

Innovation, Industry Equilibrium, and Discount Rates*

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Abstract

We develop a model to examine how discount rates affect the nature and composition of innovation within an industry. Challenging conventional wisdom, we show that higher discount rates do not discourage firm innovation when accounting for the industry equilibrium. Higher discount rates deter fresh entry—effectively acting as entry barriers—but encourage innovation through the intensive margin, which can lead to a higher industry innovation rate on net. Simultaneously, high discount rates foster explorative over exploitative innovation. Considering fluctuations in discount rates, the model further rationalizes observed patterns in innovation cyclicalities, and shows that innovation by rivals inflates incumbents' risk premia.

Keywords: Vertical and horizontal innovation, creative destruction, time-varying discount rates, risk premia.

JEL Classification Numbers: G31; G12; O31

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1 Introduction

Since [Schumpeter \(1939\)](#), scholars have argued that innovation is key to understand the real economy. In recent years, the study of the underlying determinants of corporate innovation has become particularly relevant, as firms' investment in research and development (henceforth, R&D) has increased dramatically.¹ In spite of this growing interest, the literature has so far neglected the role of discount rates in explaining firms' R&D investment. This is surprising: Existing studies show that discount rates are key to explain other corporate decisions, such as physical investment, IPO and buyout activity, or employment (i.e., [Lamont, 2000](#); [Pastor and Veronesi, 2005](#); [Haddad, Loualiche, and Plosser, 2017](#); [Hall, 2017](#)). Yet, discount rates should be arguably more significant in explaining R&D, as it is a long term investment, has an extended gestation period, and bears an uncertain outcome.

In this paper, we seek to fill this gap and develop a model to study how discount rates affect corporate innovation in industry equilibrium. Corporate finance textbooks suggest that higher discount rates should penalize cash flows expected in the far future and, thus, should discourage investments, especially longer-term ones such as R&D. Yet, this line of reasoning neglects that a firm's R&D investment largely depends on the presence and behavior of competing firms, whose decisions are also affected by discount rates. Taking into account that firms do not innovate in isolation, our model thus shows that a higher market risk premium—that is, the common component of firms' discount rates in the cross section—can indeed lead to greater innovation rates by affecting the composition and the nature of innovation within the industry—i.e., whether it is performed by incumbents or entrants, and whether it is more exploitative or explorative. Considering fluctuations in the market risk premium, the model also delivers novel insights on the cyclicity of innovation, and on the effect of competition in technology on firms' risk premia.

Our model considers an industry in which firms are subject to two sources of systematic risk: a diffusion risk that directly affects firms' cash flows, and a jump risk associated with

¹Among others, see [Doidge, Kahle, Karolyi, and Stulz \(2018\)](#); [Brown, Fazzari, and Petersen \(2009\)](#); [De Ridder \(2020\)](#).

changes in the state of the economy. The market price of risk associated with the diffusion risk is state-contingent. Consistent with the evidence in [Braguinsky, Ohyama, Okazaki, and Syverson \(2020\)](#), firms in the industry may pursue two alternative types of innovation: vertical (or explorative), which aims at major breakthroughs that improve the quality of technology; and horizontal (or exploitative), which aims at creating new products within the current technological frontier. As standard, we assume that innovation is costly and has an uncertain outcome. The industry features three types of firms: an initiator, exploiters, and entrants. The initiator represents the latest successful innovator to advance the technology frontier via a vertical breakthrough, and to start a bundle of new products that build on such breakthrough. Exploiters are firms that, exploiting the latest vertical breakthrough, have successfully developed new products via horizontal breakthroughs, and solely focus on production. Last, entrants are startups that invest in vertical and horizontal innovation with the aim of becoming initiators or exploiters. Vertical breakthroughs cast the threat of creative destruction on the initiator and exploiters, then causing their exit. Conversely, horizontal breakthroughs cause partial displacement by making some of the initiator's and exploiters' products obsolete, and thus reduce their revenues.

To disentangle the strengths at play in the model, we start by considering the case in which there is just one state of the economy and the market price of risk is constant. When abstracting from industry dynamics, we confirm the conventional wisdom that a higher market price of risk discourages a firm's R&D expenditures. However, when allowing for endogenous industry dynamics, this result is overturned. A key result of the model is that the market price of risk affects the composition of innovation within the industry. Specifically, we show that a greater market price of risk discourages entry by new firms—effectively acting as a barrier to entry—and, simultaneously, encourages innovation by active firms. Compounding these offsetting effects, the market price of risk has a non-monotonic effect on the industry-level rate at which new technologies endogenously emerge. That is, perhaps surprisingly, we show that a higher market price of risk can spur the endogenous advent of new technologies, if the ensuing higher R&D engagement of active firms (the intensive margin) more than offsets the decline in the mass of entrants (the

extensive margin). More broadly, we highlight that the market price of risk affects firms' interactions in the *technology space*, then complementing existing models that focus instead on how discount rates affect firms' strategic behavior in the product market space (i.e., [Dou, Ji, and Wu, 2020](#); [Chen, Dou, Guo, and Ji, 2020](#)).²

Our model also reveals that accounting for different types of innovation—horizontal or vertical—is key to understand how the market price of risk affects innovation incentives in equilibrium. While vertical innovation is increasing in the market price of risk due to the ensuing lower threat of creative destruction, horizontal innovation is non-monotonic due to two offsetting strengths. First, the lower rate of creative destruction associated with a higher market price of risk reduces the threat that the industry will soon experience a vertical breakthrough (and thus operate on a different technology), which encourages horizontal innovation by entrants. Second, a higher discount rate boosts innovation by the initiator, which in turn increases the threat of a vertical breakthrough and, thus, deters horizontal innovation. Overall, we show that the optimal rate of horizontal innovation decreases with the market price of risk when it is sufficiently high, in which case the second strength dominates. Hence, the model shows that a greater market price of risk stimulates the more explorative type of innovation within the industry.

We next allow the market price of risk to vary over time. We assume that the economy can be in two states, one characterized by a low market price of risk (the good state or expansion) and the other characterized by a high market price of risk (the bad state or recession), consistent with, e.g., [Lustig and Verdelhan \(2012\)](#). Our model shows that this time variation importantly affects the cyclicity of R&D pursued by different firms in the industry. We show that active firms are more R&D-intensive when the market price of risk is higher (i.e., in bad states of the economy) but, at the same time, fewer firms are active—namely, the mass of entrants investing in innovation is smaller. That is, active firms face lower competition in innovation in bad states of the economy thanks to a lower rate of creative destruction and of product obsolescence which, in turn, encourages their R&D

²[Bloom, Schankerman, and Van Reenen \(2013\)](#) show empirically that competition in technology and in the product market are two distinct types of corporate rivalry.

engagement. Moreover, whereas the aggregate (industry-level) contribution of entrants to innovation is higher in good states of the economy (i.e., when the market price of risk is lower) thanks to the greater mass of entrants, we show that the firm-level R&D investment of active entrants is actually stronger in bad states, consistent with [Howell, Lerner, Nanda, and Townsend \(2020\)](#) and [Hacamo and Kleiner \(2021\)](#). Hence, by identifying strengths steering pro- and counter-cyclicity in innovation, our paper reconciles the Schumpeterian view that firms should invest more in bad states of the economy, with the evidence that R&D investment is procyclical at the aggregate level.³ Consistent with the evidence in [Babina, Bernstein, and Mezzanotti \(2020\)](#), the procyclicity of innovation in our model is largely driven by the extensive margin.

Having analyzed how the magnitude of the market price of risk impacts the industry equilibrium, we also investigate how its fluctuations affect firms' incentives to innovate. To this end, we compare our two-state economy with an identical economy in which the market price of risk is fixed at its two-state average. We find that fluctuations in the market price of risk have the strongest impact on the extensive innovation margin. Specifically, the mass of active entrants is *larger*, on average, when allowing for these fluctuations, in which case the rate of creative destruction is greater. Consistent with the Schumpeterian view that creative destruction spurs innovation, our model shows that fluctuations in the market price of risk induce a more prominent industry turnover that, in turn, is beneficial to the endogenous advent of new technologies. That is, our model shows that fluctuations in the market price of risk are not detrimental to an industry's technological advancement.

Last, the asset pricing implications of our model show that the resolution of the idiosyncratic uncertainty associated with innovation outcomes affects firms' risk premia, consistent with [Berk, Green, and Naik \(2004\)](#) and [Gu \(2016\)](#). Unlike these earlier studies which study innovative firms in isolation, our model contributes to understand how the nature of firms' rivalry within an industry affects risk premia. While a number of papers conclude that the threat of entry by new firms in the product market makes incumbent firms *safer*,⁴ we

³See, for instance, [Griliches \(1984\)](#), [Comin and Gertler \(2006\)](#), [Barlevy \(2007\)](#), or [Fabrizio and Tsolmon \(2014\)](#), among others.

⁴See, for instance, [Bustamante and Donangelo \(2017\)](#) and [Babenko, Tserlukevich, and Boguth \(2020\)](#).

predict that stronger competition in innovation makes incumbents *riskier*. Thus, our model points out a mechanism to identify the nature of firm rivalry—either in product markets or in technology—through the impact of firm entry on expected returns, thus complementing the findings in [Bloom, Schankerman, and Van Reenen \(2013\)](#). Our model also shows that a firm’s own innovation acts as insurance against the innovation of its rivals, consistent with the patent race model by [Bena and Garlappi \(2020\)](#).

Related literature Our paper relates to the literature showing the significance of discount rates for various corporate decisions and macroeconomic dynamics (see the presidential address by [Cochrane, 2011](#)). In this strand, [Pastor and Veronesi \(2005\)](#) show that waves of initial public offerings are largely driven by declines in expected market returns. [Haddad, Loualiche, and Plosser \(2017\)](#) and [Malenko and Malenko \(2015\)](#) study the impact of discount rates on buyout activity. [Opp, Parlour, and Walden \(2014\)](#), [Dou, Ji, and Wu \(2020\)](#), and [Chen et al. \(2020\)](#) show that discount rate fluctuations affect competition in the product market—differently, our paper looks at firm’s strategic interactions in the technological space, consistent with the evidence that these are two very different types of rivalries (see, e.g. [Bloom, Schankerman, and Van Reenen, 2013](#)). Taking a macroeconomic perspective, [Hall \(2017\)](#) shows that the time variation in discount rates is a strong determinant of unemployment dynamics. We contribute to this strand by showing that the level and fluctuations of the market price of risk have a first-order impact on R&D investment, challenging the conventional wisdom that larger discount rates discourage investment.

We also contribute to the corporate finance literature studying innovation. Previous models on firms’ incentives to innovate have considered the role of managerial compensation and incentives schemes ([Manso, 2011](#)), firms’ ownership structure ([Ferreira, Manso, and Silva, 2014](#)), takeovers ([Phillips and Zhdanov, 2013](#)), financing frictions and cash availability ([Malamud and Zucchi, 2019](#); [Lyandres and Palazzo, 2016](#)), and debt financing ([Geelen, Hajda, and Morellec, 2021](#)). We look instead at the impact of discount rates on innovation

Other studies documenting a negative relation between expected returns and product market competition include [Corhay, Kung, and Schmid \(2020\)](#), [Loualiche \(2020\)](#), and [Grotteria \(2020\)](#).

in an industry equilibrium. Thus, our paper also relates to the growing empirical literature on competition in innovation—in this strand, [Kogan, Papanikolaou, Seru, and Stoffman \(2017\)](#) measure how a firm’s innovation affects its rivals; [Cunningham, Ederer, and Ma \(2021\)](#) study the role of takeovers in drug development; [Manso, Balsmeier, and Fleming \(2019\)](#) study how explorative and exploitative innovations vary over the business cycle; and [Braguinsky et al. \(2020\)](#) document how firms innovate vertically and horizontally. The predictions of our model relate closely to [Babina, Bernstein, and Mezzanotti \(2020\)](#), [Howell et al. \(2020\)](#), and [Hacamo and Kleiner \(2021\)](#), who study the cyclicalities of the innovation by incumbents and startups.

The theme of our paper also relates to earlier research showing how the study of corporate decisions in industry equilibrium improves our understanding of their underlying determinants. In this strand, [Miao \(2005\)](#) uncovers a price feedback effect by which the availability of credit may discourage entry due to its downward effect on output prices, [Hackbarth and Miao \(2012\)](#) elaborate on the link between mergers and industry dynamics, and [Pindyck \(2009\)](#) characterizes how risk affects firms’ incentives to enter product markets. Closely related to [Pindyck \(2009\)](#), we study how discount rates affect innovation in the intensive and extensive margin. Moreover, [Bustamante and Donangelo \(2017\)](#) find that industries with higher exposure to systematic risk are less attractive to new entrants and remain more concentrated, given that the cash flows generated by incumbents are discounted at higher rates. We show instead that discount rates act as a barrier to entry into a firm’s *technology space*, and affect the composition of innovation within an industry.

Last, while a growing number of papers have documented that firms in less competitive product markets with higher markups also have higher expected returns (i.e., [Corhay, Kung, and Schmid, 2020](#); [Loualiche, 2020](#); [Babenko, Tserlukevich, and Boguth, 2020](#)), fewer papers have focused on studying the impact of competition in technology (innovation) on risk premia. As relevant exceptions, [Bena and Garlappi \(2020\)](#) consider a patent race model of two firms in which the expected return of one firm decreases with its own innovation output and increases with that of its rival, and [Grotteria \(2020\)](#) studies how lobbying relates to

innovation and risk premia in a model of endogenous innovation.⁵

The paper is organized as follows. Section 2 presents the model. Section 3 analyzes the model implications when the market price of risk is constant, whereas Section 4 allows for time-variation in the market price of risk. Section 5 analyzes the model’s asset pricing implications. Section 6 concludes. Technical developments are gathered in the Appendix.

2 The model

The economic environment Time is continuous, and the horizon is infinite. We consider a cluster of firms, or industry, which compete in innovation. Firms are subject to two sources of aggregate risk: a diffusion risk and a jump risk. These risks are both priced and affect the dynamics of the stochastic discount factor, denoted by ξ_t , which satisfies the following jump-diffusion process:

$$\frac{d\xi_t}{\xi_t} = -r dt - \eta(j_{t-}) d\tilde{B}_t + \sum_{j_t \neq j_{t-}} (e^{\theta(j_{t-}, j_t)} - 1) d\tilde{N}_t^{(j_{t-}, j_t)}. \quad (1)$$

In this equation, r is the constant risk-free rate of the economy. $d\tilde{B}_t$ is a standard Brownian motion representing the systematic source of diffusion risk, and $\eta(j_t)$ represents the associated market price of risk. $\tilde{N}_t^{(j_{t-}, j_t)}$ is a compensated Poisson process with intensity $\tilde{\pi}_{j_{t-}}$, whereas $\theta(j_{t-}, j_t)$ represents the associated risk adjustment.

The jump risk represents switches in the state of the economy. For simplicity, we assume that the economy can be in two states, denoted by $j = G, B$: a good (expansion) state $j = G$ and a bad (recession) state $j = B$. We assume that the market price of the diffusion risk is state-contingent so that $\eta(j_t) \equiv \eta_j$, and we impose $\eta_G < \eta_B$ —i.e., the market price of risk is countercyclical, as documented by [Lustig and Verdelhan \(2012\)](#) among others.⁶ A

⁵More broadly, our paper relates to [Garleanu, Kogan, and Panageas \(2012\)](#), [Garleanu, Panageas, and Yu \(2012\)](#), [Kogan, Papanikolaou, and Stoffman \(2020\)](#), [Kung and Schmid \(2015\)](#), and [Bena, Garlappi, and Gruning \(2016\)](#), studying how the interaction between innovation and consumption affect asset prices in general equilibrium.

⁶As shown by [Campbell and Cochrane \(1999\)](#), the countercyclicity of the market price of risk can be driven by, e.g., time-varying risk aversion.

switch in the state of the economy causes a jump in the stochastic discount factor, meaning that investors require a compensation for the risk of the economy switching states. This compensation translates into a wedge between the transition intensity under the physical probability measure and under the risk neutral measure. Using the risk adjustment $\theta(j_{t-}, j_t) \equiv \theta_j$, the risk-neutral transition intensities satisfy $\pi_j = e^{\theta_j} \tilde{\pi}_j$ in each state. As in [Bolton, Chen, and Wang \(2013\)](#), we assume that $\theta_G = -\theta_B > 0$, which implies that the transition intensity from state G (respectively, B) to state B (G) is higher (smaller) under the risk-neutral probability measure than under the physical one. In other words, risk averse agents expect the good (bad) state to be shorter (longer).

Innovation types We follow [Howitt \(1999\)](#) and acknowledge that firms within the industry may pursue two types of innovations: Vertical (or explorative) or horizontal (or exploitative). Consistent with [Arora, Belenzon, and Sheer \(2021\)](#), vertical innovations are more related to “research,” whereas horizontal innovations are more related to “development.” Specifically, vertical innovations represent major breakthroughs in the quality of technology, denoted by q_t . Conversely, horizontal innovations build on the existing technological quality (i.e., on the latest vertical breakthrough) and aim at creating new products (or varieties). Hence, horizontal innovations can be seen as follow-up applications of the current technology aimed at creating new product lines. In the spirit of [Howitt \(1999\)](#), follow-up horizontal innovations applied to a given quality level q_t eventually run into diminishing returns to scale. That is, horizontal innovations become gradually less productive until a new vertical breakthrough initiates a new technological cluster that makes horizontal innovation productive again. We define a technological cluster as the collection of new products that stem from a given increase in the quality of technology.

We assume that the industry features three different types of firms: an initiator, exploiters, and entrants. The initiator, denoted by the value function U_j in each state j of the economy, represents the latest vertical innovator upgrading quality q_t , then starting a new technological cluster and creating a mass of new products. The initiator continues to invest in innovation while producing and selling these products. The exploiters, denoted

by the value function X_j in each state j , are firms that have successfully developed new product lines through horizontal innovation within a given technological cluster, and solely focus on production (i.e., they do not further invest in R&D). Entrants, denoted by the value function W_j , are startups on the sideline and represent the industry’s wellspring. Entrants invest in vertical and horizontal innovation and have the potential to become the new initiator (and start a new technological cluster) or an exploiter (then introducing new products). We next describe these firm types in detail.

Initiator (innovating producer) The latest vertical innovator improving the industry’s quality level q_t becomes the initiator of a new technological cluster and drives the existing producers (that is, the previous initiator and exploiters) out of the market. Using this novel technology, the initiator manufactures a mass M_t of products. In each product line i , the firm faces the following demand function:

$$p_{it} = \Gamma_j \left(\frac{Y_{it}}{q_t} \right)^{-\beta}, \quad (2)$$

where p_{it} represents the selling price associated to product i , Y_{it} represents the associated quantity, and $\beta \in (0, 1)$ is the inverse of the price elasticity of demand.⁷ Γ_j represents a demand-shift parameter, which varies with the state of the economy j . We assume that the cost of production is normalized to one in all product lines, without loss of generality—thus, $p_{it} - 1$ represents the markup in product line i . Following previous literature, we assume that all product lines exhibit the same demand function, and each product line is a monopoly until an entrant attains a breakthrough, as we explain below.⁸

The initiator of the technological cluster earns revenues from producing the M_t goods and, at the same time, continues to invest in innovation. We denote by z_t the initiator’s

⁷As we express p and Y as a function of time (as captured by the subscript t), we omit their dependence to the state of the economy (subscript j).

⁸The assumption that each product line represents a monopoly is standard in this literature, see e.g., Klette and Kortum (2004), Aghion and Howitt (1992), Howitt (1999), Acemoglu, Akcigit, Alp, Bloom, and Kerr (2018), Akcigit and Kerr (2018), among many others. Because all the product lines face the same demand function, we drop the subscript i in the following.

innovation intensity at time t . We follow previous literature in capturing the key features of investment in innovation: It is costly and has an uncertain outcome. That is, if the firm bears the following flow cost

$$\Phi(z, q, M) = \zeta \frac{z_t^2}{2} q_t M_t, \quad \zeta > 0, \quad (3)$$

it attains a breakthrough at Poisson rate ϕz_t , where ϕ is a positive constant. This specification implies that the occurrence rate of such events is more likely if the firm spends more on innovation. It also captures the idea that innovation becomes more costly if quality is greater or if the mass of current product lines is larger.⁹ We assume that when the initiator attains a breakthrough, quality jumps by a factor $\lambda > 1$ and the mass of product lines jumps by a factor $\varphi > 1$. That is, because the initiator has specific “working” knowledge of the particular industry, an innovation increasing the quality of technology results in the creation of new products, in the spirit of Nelson (1959) and Akcigit, Hanley, and Serrano-Velarde (2021).

Absent breakthroughs by other firms, the cash flows of the initiator satisfy the following dynamics:

$$\begin{aligned} dC_t &= [Y_t(p_t - 1) M_t - \Phi(z, q, M)] dt + \sigma Y_t M_t d\tilde{B}_t^U \\ &= [Y_t(p_t - 1) M_t - \Phi(z, q, M)] dt + \sigma Y_t M_t \left[\rho d\tilde{B}_t + \sqrt{1 - \rho^2} d\tilde{B}_t^{U\perp} \right]. \end{aligned} \quad (4)$$

The first term represents the initiator’s profits from production in the M_t product lines net of R&D expenditures. Throughout our analysis, we focus on cases in which this term is positive, so to avoid the degenerate case in which the initiator always makes losses in expectation. The second term represents the volatility of the initiator’s cash flows, which increases with the firm’s production rate. The parameter σ is a positive constant, and \tilde{B}^U is a standard Brownian motion under the physical probability measure. The Brownian

⁹Scalability of the innovation costs in quality or product lines is consistent with previous models of endogenous growth, see Aghion, Akcigit, and Howitt (2014) for a survey as well as Akcigit, Hanley, and Serrano-Velarde (2021) or Acemoglu et al. (2018) for recent contributions.

motion \tilde{B}^U is correlated with the aggregate shock \tilde{B} by a factor $\rho \geq 0$. That is, \tilde{B}^U can be decomposed into the orthogonal components \tilde{B}_t and $\tilde{B}_t^{U\perp}$ through ρ , where $\tilde{B}_t^{U\perp}$ captures idiosyncratic risk independent of the aggregate (priced) risk \tilde{B}_t .

Consistent with the evidence in [Argente, Lee, and Moreira \(2020\)](#), we assume that the initiator loses some of its product lines if an entrant successfully attains a horizontal innovation. In this case, new product lines are launched, which make some of the initiator's existing products obsolete. Namely, if an entrant attains a horizontal innovation creating a mass ωM_{t-} of new products, the initiator's product lines drop from M_{t-} to $M_t = M_{t-}(1 - \omega\delta)$, and so do cash flows.¹⁰ The parameter $\delta \in (0, 1]$ represents the degree of overlap between these new products and those of the initiator. A greater overlap means that more of the initiator's products become obsolete due to a horizontal breakthrough. We denote by Ψ_{ht} the endogenous rate of horizontal displacement in the industry—equivalently, the rate at which entrants as a whole attain horizontal breakthroughs.

In addition to obsolescence due to horizontal innovation, the initiator loses all of its product lines if an entrant takes over its technological leadership via vertical innovation and starts a new technological cluster. In this case, the initiator is hit by creative destruction. We denote the endogenous rate of creative destruction—i.e., the rate at which entrants attain a vertical breakthrough—by Ψ_{vt} . When creative destruction hits, the initiator liquidates its assets and exits. We assume that liquidation is costly, as the initiator recovers just a fraction $\alpha \in [0, 1)$ of its value.

Entrants There is a continuum of entrant firms on the sideline, whose endogenous mass is denoted by μ . Entrants only invest in innovation and can be interpreted as startups. Because entrants do not have ongoing production—i.e., differently from the initiator, they do not have working knowledge of specific product lines—they do not benefit from the synergy between vertical and horizontal R&D as the initiator does. Thus, entrants need to spend on vertical or horizontal innovation separately.

We denote an entrant's innovation rate targeting vertical breakthroughs by v_t at any

¹⁰As shown by Equation (4), the cash flows of the initiator scale up with its mass of product lines M_t .

time t . Similar to the initiator, v_t governs the Poisson rate of vertical breakthroughs—given by $\phi_v v_t$ with ϕ_v being a positive constant—and entails the flow cost:

$$\Phi_v(v, q, M) = \zeta_v \frac{v_t^2}{2} q_t M_t, \quad \zeta_v > 0. \quad (5)$$

When the entrant attains a vertical breakthrough, the industry’s technological quality jumps by a factor $\Lambda > 1$, a new technological cluster is created, and the entrant takes over the initiator’s industry leadership. In turn, we denote by h_t an entrant’s innovation rate targeting horizontal breakthroughs. When spending the amount

$$\Phi_h(h, q, M) = \zeta_h \frac{h_t^2}{2} q_t M_t, \quad \zeta_h > 0, \quad (6)$$

an entrant attains a horizontal innovation at a Poisson rate $\phi_h h$, where ϕ_h is a positive parameter. That is, the greater the rate h_t , the more likely the entrants will attain a horizontal breakthrough creating a mass of new products $M_{Xt} = \omega M_{t-}$, where $\omega \in [0, 1]$ is a constant and M_{t-} represents the mass of the initiator’s product lines right before the breakthrough. As M_t decreases as successive horizontal innovations are introduced within a given technological cluster, we acknowledge that horizontal innovations run into diminishing returns to scale (see, e.g., [Howitt, 1999](#)). Once an entrant attains a horizontal breakthrough, it becomes an exploiter thereafter.¹¹

Entrants do not have ongoing production, and their risky investment in innovation exposes them to random shocks—for instance, random outflows or windfalls in the development of new ideas or products. Specifically, we assume that entrants’ cash flows are described by the process:

$$dC_t^W = \left[-\frac{1}{2} (\zeta_v v_t^2 + \zeta_h h_t^2) dt + \sigma_W d\tilde{B}_t^W \right] M_t q_t \quad (7)$$

where σ_W is a positive constant. \tilde{B}_t^W is a standard Brownian motion under the physical

¹¹Notably, because entrants aim to improve on the initiator’s technology and products, their innovation cost is a function of current quality q_t and of the initiator’s mass of product lines M_t .

probability measure, which is correlated with the aggregate shock \tilde{B}_t by a factor $\rho_W \geq 0$.¹² This specification implies that entrants have negative cash flows in expectation, consistent with the evidence that startups usually lack steady revenues while developing an innovative idea. As entrants aim at improving the quality and expand the bundle of products launched by the initiator, their innovation costs and volatility depend on q_t and M_t .

At the outset, entrants face an entry cost $K_t = \kappa q_t M_t$ to start investing in innovation, which can then be interpreted as the cost of installing the firm's technological capital. The magnitude of the cost K_t varies over time due to technological improvements in q_t or due to the expansion or contraction of M_t . As a result, if the initiator or other entrants attain a breakthrough, an entrant needs to adjust its technological capital in proportion to the ensuing change in q_t and/or M_t , consistent with [Luttmer \(2007\)](#).

Exploiters (non-innovating producers) Entrants who successfully attain a horizontal breakthrough—i.e., who create a mass of new products M_{Xt} —become the monopolistic producers in such new product lines. These firms, which we refer to as exploiters, give up on innovation and simply maximize their value by choosing the production quantity Y_{Xt} and the selling price p_{Xt} in their product lines. As the initiator, the exploiters face the demand function (2) in each product line. An exploiter's cash flows are given by:

$$dC_t^X = Y_{Xt} (p_{Xt} - 1) M_{Xt} dt + \sigma_X Y_{Xt} M_{Xt} d\tilde{B}_t^X. \quad (8)$$

In this equation, σ_X is a positive constant, and \tilde{B}_t^X is a standard Brownian motion under the physical probability measure that is correlated with the aggregate Brownian shock \tilde{B}_t by a factor $\rho_X \geq 0$.¹³ Because the initiator and the exploiters both produce goods in the same industry, we assume that their exposure to aggregate risk is the same, i.e., $\rho_X = \rho$. As for the initiator, an exploiter's cash flow volatility increases with its production rate.

¹²As for the initiator, we can decompose the Brownian motion \tilde{B}_t^W into the systematic source of risk and an orthogonal component, representing purely idiosyncratic risk.

¹³As for the other firms, the Brownian motion \tilde{B}_t^X can be decomposed into the orthogonal components \tilde{B}_t and $\tilde{B}_t^{X\perp}$ through ρ_X , where $\tilde{B}_t^{X\perp}$ is independent to the aggregate (priced) risk \tilde{B}_t .

Similar to the initiator, the exploiters can lose product lines due to the obsolescence triggered by subsequent horizontal breakthroughs by entrants. When this happens, the exploiters lose a fraction $\omega\delta$ of their active product lines. In addition, exploiters face the threat of creative destruction. That is, exploiters are driven out of the market if a vertical breakthrough improves the current quality q_t and starts a new technological cluster. When creative destruction hits, the exploiters liquidate and recover just a fraction $\alpha_X \in [0, 1]$ of their value, similar to the initiator. Notably, the exploiters are subject to the threat of exit when either the initiator or the entrants attain a vertical breakthrough.

Industry equilibrium We consider an industry equilibrium in which: (1) the initiator maximizes its value by choosing its optimal production and innovation rate; (2) exploiters maximize their value by choosing their optimal production rate; (3) entrants maximize their value by choosing their optimal vertical and horizontal innovation rates; (4) the mass of active entrants makes the free-entry condition binding at any time.

As we show in the following, the equilibrium rate of creative destruction Ψ_{vt} is derived endogenously as the aggregate rate at which active entrants attain a vertical breakthrough starting a new technological cluster. In turn, the equilibrium rate of horizontal displacement Ψ_{ht} is the aggregate rate at which active entrants attain a horizontal breakthrough, then creating new products and causing obsolescence of existing products. Notably, Ψ_{vt} and Ψ_{ht} affect the initiator's and the exploiters' value by casting the threat of exit or of product obsolescence and, in turn, are affected by the initiator's and the exploiters' value through their impact on the entrants' optimal innovation rate.

3 Constant market price of risk

To disentangle the forces at play, we start by considering the case in which there is only one state of the economy, in which the market price of risk is constant and denoted by η .¹⁴

¹⁴As there is just one state, the demand shift parameter $\Gamma_j = \Gamma$ is constant too.

3.1 Model solution

Using Girsanov theorem, the initiator's cash flow process satisfies the following dynamics under the risk neutral measure:

$$dC_t = Y_t (p_t - 1 - \sigma\rho\eta) M_t dt - \Phi(z_t, q_t, M_t) dt + \sigma Y_t M_t dB_t^U, \quad (9)$$

where B_t^U is the Brownian motion describing the initiator's shocks under the risk-neutral measure. Because the optimal production quantity affects the volatility of revenues in each product line, the risk-adjustment is proportional to Y_t and M_t . Using standard arguments, the value of the initiator satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned} rU(q, M) = \max_{z, Y} & MY(p - 1 - \sigma\eta\rho) - \zeta \frac{z^2}{2} qM + \phi z [U(\lambda q, \varphi M) - U(q, M)] \\ & + \Psi_v [\alpha U(q, M) - U(q, M)] + \Psi_h [U(q, M(1 - \omega\delta)) - U(q, M)]. \end{aligned} \quad (10)$$

The term on the left-hand side of this equation is the return required by risk-neutral investors. The right-hand side is the expected change in firm value on an infinitesimal time interval. Namely, the first two terms are the risk-adjusted expected cash flows net of R&D expenditures. The third term is the expected change in firm value due to a technological breakthrough by the initiator, which triggers an increase in quality by λ and an expansion in the mass of product lines by φ . The fourth term represents the effect of creative destruction triggered by entrants' vertical innovations (occurring at rate Ψ_v), in which case the initiator exits and recovers just a fraction α of firm value. The last term is the effect of obsolescence triggered by entrants' horizontal innovations (occurring at rate Ψ_h), which erodes a fraction $\omega\delta$ of the initiator's product lines.

We conjecture that the value of the initiator scales with quality q_t and with the mass of product lines M_t , $U(q_t, M_t) = q_t M_t u$, where u represents the initiator's scaled value. Also, we define by $y \equiv Y_t/q_t$ the production quantity in each product line scaled by quality.

Substituting these definitions into equation (10) gives:

$$ru = \max_{z,y} y^{1-\beta} \Gamma - y - \sigma \eta \rho y - \frac{z^2}{2} \zeta + \phi z (\lambda \varphi - 1) u - \Psi_v u (1 - \alpha) - \Psi_h u \omega \delta. \quad (11)$$

The optimization problem of the initiator is over the production quantity y and over the innovation decision z . Differentiating the above equation with respect to y gives the optimal production quantity per product line and the associated selling price:

$$y(\eta) = \left(\frac{\Gamma(1-\beta)}{1+\sigma\eta\rho} \right)^{\frac{1}{\beta}} \Rightarrow p(\eta) = \Gamma y^{-\beta} = \frac{1+\sigma\eta\rho}{1-\beta}. \quad (12)$$

As illustrated by equation (4), the firm endogenously chooses its exposure to aggregate risk by setting its optimal production quantity. Equation (12) shows that if the market price of risk η is greater, the firm reduces its optimal production quantity and increases the selling price. That is, the firm effectively reduces its exposure to aggregate risk and, by increasing the selling price, it passes the higher price of risk on to the consumers.

Differentiating equation (11) with respect to z gives the optimal innovation rate:

$$z(\eta) = \frac{\phi}{\zeta} (\lambda \varphi - 1) u(\eta). \quad (13)$$

This expression suggests that the higher the value of the initiator, the greater its innovation rate, as the surplus from attaining a technological breakthrough widens. Moreover, the optimal innovation rate increases if R&D expenditures are more likely to translate into technological breakthroughs (higher ϕ), if the returns to innovation are greater (larger λ or φ), or if innovation is less costly (ζ is smaller). By substituting equations (12) and (13) into equation (11) gives

$$\frac{\phi^2}{2\zeta} (\lambda \varphi - 1)^2 u^2 - (r + \Psi_v(1 - \alpha) + \Psi_h \omega \delta) u + \Upsilon(\eta) = 0. \quad (14)$$

where $\Upsilon(\eta) \equiv \beta \left(\frac{1-\beta}{1+\sigma\eta\rho} \right)^{\frac{1}{\beta}-1} \Gamma^{\frac{1}{\beta}}$ represents the initiator's risk-adjusted profits from produc-

tion, which decrease with the market price of risk.

Consider now the dynamics of the exploiters.¹⁵ Recall that exploiters are entrants that have attained a horizontal breakthrough creating a mass of new product lines $M_{Xt} = \omega M_{t-}$. We define $y_X \equiv Y_{Xt}/q_t$ as an exploiter's production quantity per active product line scaled by quality. Exploiter value then satisfies:

$$\begin{aligned} rX(q, M_X) = \max_{Y_X} & M_X Y_X (p_X - 1 - \eta\rho\sigma_X) + (\Psi_v + \phi z) (\alpha_X X(q, M_X) - X(q, M_X)) \\ & + \Psi_h [X(q, (1 - \omega\delta)M_X) - X(q, M_X)]. \end{aligned}$$

As for equation (10), the right-hand side is the expected change in exploiter value over an infinitesimal time interval. The first-term represents the exploiter's risk-adjusted expected profits. The second term represents the effect of vertical innovations by entrants (occurring at rate Ψ_v) or by the initiator (occurring at rate ϕz), which trigger the advent of a new technological cluster and cause the exit of the incumbent exploiters. The third term represents the effect of horizontal innovations by entrants (occurring at rate Ψ_h), which cause the exploiters to lose a fraction of their product lines. Exploiters maximize their value by choosing their optimal production quantity Y_X .

We conjecture that the exploiter value function satisfies $X(q_t, M_{Xt}) = q_t M_{Xt} \tilde{x}$, where \tilde{x} represents the exploiter value scaled by the industry's quality level q_t and by the mass of its active product lines M_{Xt} . When an exploiter starts production, its mass of product lines can be expressed as a function of the product lines of the initiator: $M_{Xt} = \frac{M_t}{1-\omega\delta}\omega$.¹⁶ Thus, we can express the exploiter value as a function of the active product lines of the initiator M_t as follows:

$$X(q_t, M_{Xt}) = q_t M_{Xt} \tilde{x} = q_t M_t \tilde{x} \frac{\omega}{1 - \omega\delta} = q_t M_t x. \quad (15)$$

¹⁵The dynamics of exploiters' and entrants' cash flows under the risk neutral measure are reported in Appendix A.1.

¹⁶Recall that right after a horizontal breakthrough leading to the emergence of a new exploiter, the initiator's product lines are $M_t = M_{t-}(1 - \omega\delta)$.

To make the scaled value of the exploiter comparable to the other scaled quantities, we define $x = \tilde{x} \frac{\omega}{1-\omega\delta}$, which satisfies:

$$rx = \max_{y_X \geq 0} \frac{\omega}{1-\omega\delta} y_X \left(\Gamma y_X^{-\beta} - 1 - \eta\rho\sigma_X \right) - (\phi z + \Psi_v)(1 - \alpha_X)x - \Psi_h\omega\delta x. \quad (16)$$

Maximizing this equation with respect to y_X gives

$$y_X(\eta) = \left(\frac{\Gamma(1-\beta)}{1+\eta\rho\sigma_X} \right)^{\frac{1}{\beta}} \quad (17)$$

and the associated selling price is $p_X = \frac{1+\sigma_X\eta\rho}{1-\beta}$. Notably, the initiator and the exploiters choose a different production quantity due to the difference in their cash flow volatilities (see equation (12)), which in turn results in a different exposure to systematic risk. Substituting equation (17) into equation (16) gives the valuation equation for the exploiter:

$$rx = \frac{\beta\omega}{1-\omega\delta} \left(\frac{1-\beta}{1+\sigma_X\rho\eta} \right)^{\frac{1}{\beta}-1} \Gamma^{\frac{1}{\beta}} - (\phi z + \Psi_v)x(1 - \alpha_X) - \Psi_h\omega\delta x. \quad (18)$$

Next, we study the dynamics of entrants, whose value $W(q, M)$ is a function of the current quality level and the product lines it tries to improve on. Entrant value satisfies the following HJB equation:

$$\begin{aligned} rW(q, M) = \max_{v, h} & -qM \left(\eta\rho_W\sigma_W + \frac{\zeta_v}{2}v^2 + \frac{\zeta_h}{2}h^2 \right) + \phi_v v [U(\Lambda q, M) - W(q, M)] \\ & + \phi_h h [X(q, \omega M) - W(q, M)] + \phi z [W(\lambda q, \varphi M) - W(q, M) - K(\lambda\varphi - 1)] \\ & + \Psi_v^- [W(\Lambda q, M) - W(q, M) - K(\Lambda - 1)] + \Psi_h^- [W(q, M(1 - \omega\delta)) - W(q, M) + K\omega\delta]. \end{aligned} \quad (19)$$

The first term on the right-hand side represents an entrant's risk-adjusted expected outflow on any time interval. The second term represents the effect of a vertical breakthrough by the entrant occurring at rate $\phi_v v$, in which case it starts a new technological cluster and becomes the new incumbent initiator. The third term represents the effect of a horizontal breakthrough by the entrant occurring at rate $\phi_h h$, in which case the entrant becomes an

exploiter. The fourth, fifth, and sixth terms represent the effect of breakthroughs by the current initiator (occurring at rate ϕz), vertical breakthroughs by other active entrants (occurring at rate Ψ_v^-), or horizontal breakthroughs by other active entrants (occurring at rate Ψ_h^-), respectively. The fourth and fifth terms mean that, whenever the initiator or other entrants attain vertical breakthroughs, the entrant needs to catch up with the new technology, consistent with [Luttmer \(2007\)](#). Catching up on technology requires an upgrade cost related to the size of the breakthrough—depending on whether the vertical breakthrough is attained by the initiator or by an entrant, it is, respectively, $K(\lambda\varphi - 1)$ or $K(\Lambda - 1)$. Conversely, as in [Howitt \(1999\)](#), horizontal breakthroughs erode the value of the initiator and of the exploiters, as their product lines are competed away. It follows that the entrants' perspective earnings from innovation fall, and the entrant responds by adjusting its capital downwards by $K\omega\delta$, as illustrated by the last term in equation (19).

As for the other firms in the model, we conjecture that the entrant value scales with $M_t q_t$, i.e., $W(q_t, M_t) = w q_t M_t$ where we denote by w the scaled value of a perspective entrant. Using this property, equation (19) boils down to

$$\begin{aligned}
rw = \max_{v,h} & -\eta\rho_W\sigma_W - \frac{\zeta_v}{2}v^2 - \frac{\zeta_h}{2}h^2 + \phi_v v [\Lambda u - w] + \phi_h h [\omega x - w] \\
& + \phi z (\lambda\varphi - 1) (w - \kappa) + \Psi_v^- (\Lambda - 1) (w - \kappa) - \Psi_h^- (w - \kappa)\omega\delta.
\end{aligned} \tag{20}$$

Differentiating this equation with respect to v gives the optimal vertical innovation rate of entrants:

$$v(\eta) = \frac{\phi_v}{\zeta_v} (\Lambda u(\eta) - w). \tag{21}$$

This equation suggests that the entrants' incentives to engage in vertical innovation increase if the value of the initiator is greater—in which case the “reward” upon attaining a vertical breakthrough is more attractive. In turn, differentiating equation (20) with respect to h gives the optimal horizontal innovation rate:

$$h(\eta) = \frac{\phi_h}{\zeta_h} (\omega x(\eta) - w). \tag{22}$$

This equation implies that the entrants' incentives to engage in horizontal innovation increase as exploiters are more valuable—in which case the “reward” upon a horizontal breakthrough is more attractive.

Creative destruction is defined as the rate at which entrants take over the initiator's leadership. Aggregating the rate of vertical innovation across the mass of active entrants μ thus gives the rate of creative destruction:

$$\Psi_v(\eta) = \mu(\eta) \phi_v v(\eta). \quad (23)$$

In turn, the rate of horizontal displacement obtains by aggregating the rate of horizontal innovation across the mass of active entrants:

$$\Psi_h(\eta) = \mu(\eta) \phi_h h(\eta). \quad (24)$$

In these expressions, $\mu(\eta)$ is endogenously determined such that the free-entry condition $w = \kappa$ is binding. Lastly, we pin down the aggregate rate at which new technological clusters endogenously arise, which we denote by $\mathcal{I}(z, \Psi_v)$. Because both initiator and entrants contribute to vertical innovation aimed at starting new technological clusters, $\mathcal{I}(z, \Psi_v)$ satisfies

$$\mathcal{I}(z, \Psi_v) = \phi z(\eta) + \Psi_v(\eta) = \underbrace{\frac{\phi^2}{\zeta} u(\eta)(\lambda\varphi - 1)}_{\text{Initiator}} + \underbrace{\mu(\eta) \frac{\phi_v^2}{\zeta_v} [\Lambda u(\eta) - \kappa]}_{\text{Entrants}} \quad (25)$$

The first term represents the contribution of the initiator to the aggregate vertical innovation rate, whereas the second term is the contribution of entrants. Next, we investigate the properties of the industry equilibrium.

3.2 Model analysis

We now analyze the model solution. To disentangle the strengths at play, we start by considering simpler cases for which we obtain analytical results. We first analyze the case in which the rate of creative destruction and the rate of horizontal obsolescence are exogenous. This case is akin to considering firms in isolation, instead of studying them in the industry equilibrium. As a second step, we allow for endogenous industry dynamics in two corner cases: an industry in which entrants engage in vertical innovation only, and an industry in which entrants invest exclusively in horizontal innovation.

3.2.1 Exogenous industry dynamics

Suppose that the rate of creative destruction Ψ_v and the rate of horizontal displacement Ψ_h are exogenous and constant. In this case, the value of the initiator continues to satisfy equation (11), but Ψ_v and Ψ_h are insensitive to the market price of risk η . Solving equation (14) for given Ψ_v and Ψ_h yields the value of the initiator:

$$u(\eta) = \frac{r + \Psi_v(1 - \alpha) + \Psi_h\omega\delta - \sqrt{(r + \Psi_v(1 - \alpha) + \Psi_h\omega\delta)^2 - 2\Upsilon(\eta)\frac{\phi^2}{\zeta}(\lambda\varphi - 1)^2}}{\phi^2(\lambda\varphi - 1)^2\zeta^{-1}}. \quad (26)$$

The next proposition follows (see Appendix A.1.1 for a proof).

Proposition 1 *For exogenous Ψ_v and Ψ_h , the initiator's innovation rate satisfies:*

$$z(\eta) = \frac{r + \Psi_v(1 - \alpha) + \Psi_h\omega\delta - \sqrt{(r + \Psi_v(1 - \alpha) + \Psi_h\omega\delta)^2 - 2\Upsilon(\eta)\frac{\phi^2}{\zeta}(\lambda\varphi - 1)^2}}{\phi(\lambda\varphi - 1)}, \quad (27)$$

which is a decreasing function of the market price of risk η .

By abstracting from endogenous industry dynamics—i.e., neglecting that Ψ_v and Ψ_h are themselves functions of η in equilibrium—Proposition 1 shows that a greater market price of risk leads to a lower innovation rate. In fact, by decreasing the expected profits from production—and, thus, the expected surplus from innovation—a greater market price

of risk decreases the initiator's optimal investment in innovation. This result is in line with the received wisdom that a greater market price of risk depresses long-term investment such as R&D.

Consider next the value of exploiters when Ψ_v and Ψ_h are exogenous. Solving equation (18) with respect to x gives:

$$x(\eta) = \frac{1}{[r + (\phi z + \Psi_v)(1 - \alpha_X) + \Psi_h \omega \delta]} \frac{\beta \omega}{1 - \omega} \left(\frac{1 - \beta}{1 + \sigma_X \rho \eta} \right)^{\frac{1}{\beta} - 1} \Gamma^{\frac{1}{\beta}}. \quad (28)$$

This equation illustrates that the exploiter value decreases with Ψ_v and Ψ_h , as creative destruction and horizontal displacement respectively cause exit and an erosion in their profits through product line obsolescence. When these quantities are exogenous, the value of the exploiter decreases with η , as it is for the initiator. However, when Ψ_v , and Ψ_h are endogenous, the net impact of η on x is more nuanced, as we show next.

3.2.2 Endogenous industry dynamics in corner cases

We now focus on two corner cases featuring endogenous industry dynamics (i.e., Ψ_v or Ψ_h are endogenous).

Entrants only invest in vertical innovation If entrants invest in vertical innovation only, the industry features two types of firms: the initiator and the entrants (i.e., there are no exploiters as entrants do not pursue horizontal innovation).¹⁷ In this case, we can solve for the value of the initiator in closed form, which satisfies the following expression (see Appendix A.1.2):

$$u(\eta) = \frac{1}{\Lambda} \left(\kappa + \frac{\sqrt{2\zeta_v(r\kappa + \eta\rho_W\sigma_W)}}{\phi_v} \right). \quad (29)$$

Notably, the value of the initiator is an increasing function of the entrants' exposure to aggregate risk $\eta\rho_W\sigma_W$. Recall that the industry equilibrium requires that entrant value equals the entry cost κ due to free entry. To offset a greater discounting due to a larger η —

¹⁷That is, the initiator is subject to creative destruction, but there is no horizontal displacement.

which should push the entrant value down—the surplus from innovation needs to increase through an increase in the value of initiators, and the mass of active entrants adjusts accordingly—then pushing entrant value up. The next proposition illustrates the sensitivity of the endogenous equilibrium quantities to η (see Appendix A.1.2).

Proposition 2 *When entrants only invest in vertical innovation, the innovation rate of the initiator satisfies:*

$$z(\eta) = \frac{\phi(\lambda - 1)}{\zeta\Lambda} \left[\kappa + \sqrt{\frac{2\zeta_v(r\kappa + \eta\rho_W\sigma_W)}{\phi_v^2}} \right] \quad (30)$$

and the vertical innovation rate of active entrants satisfies:

$$v(\eta) = \sqrt{\frac{2(r\kappa + \eta\rho_W\sigma_W)}{\zeta_v}}. \quad (31)$$

Both $z(\eta)$ and $v(\eta)$ increase with η . At the same time, the mass of active entrants $\mu(\eta)$ as well as the rate of creative destruction $\Psi_v(\eta)$ decrease with η .

In sharp contrast with Proposition 1 (in which the initiator is considered in isolation so that the rate of creative destruction Ψ_v is exogenous), Proposition 2 shows that the optimal innovation rate of the initiator $z(\eta)$ and of active entrants $v(\eta)$ *increase* with the market price of risk when accounting for the industry equilibrium (and, thus, deriving Ψ_v endogenously). Thus, the initiator’s contribution to the industry’s innovative advancement increases with η . At the same time, Proposition 2 shows that a greater market price of risk has a negative impact on the extensive innovation margin, leading to a reduction in the mass of active entrants. That is, η effectively acts as an entry barrier. Put together, these results imply that a higher η bears two offsetting effects on the overall entrants’ contribution to innovation. First, active entrants invest more in innovation when the market price of risk is higher—in fact, v increases with η . Second, the mass of active entrants shrinks if η is greater. Proposition 1 illustrates that the second strength dominates and, thus, the rate of creative destruction Ψ_v decreases with η . As a result, the incumbent initiator is less

threatened by competition in innovation and, therefore, is more valuable and has greater incentives to spend on R&D as η rises.

Entrants only engage in horizontal innovation We next consider the case in which entrants only pursue horizontal innovation. As in the full model, there are three types of firms in the economy: initiator, entrants, and exploiters. However, differently from the full model, the initiator is not subject to creative destruction but to horizontal displacement only—i.e., the initiator is infinitely-lived in this case. Entrants attaining a horizontal breakthrough become exploiters, whose value satisfies the following expression:

$$x(\eta) = \frac{1}{\omega} \left(\kappa + \frac{\sqrt{2\zeta_h(r\kappa + \eta\rho_W\sigma_W)}}{\phi_h} \right). \quad (32)$$

Differently from equation (28) in which the exploiter is considered in isolation (so that the rate of horizontal displacement Ψ_v is exogenous), equation (32) suggests that the exploiter value increases with η when Ψ_v is endogenous. As in the case with vertical innovation only, the industry equilibrium requires that entrant value equals the entry cost for the free-entry condition to be satisfied. Thus, to offset the value-decreasing effect of a greater market price of risk, the surplus from horizontal innovation needs to increase through an increase in the value of exploiters and an adjustment in the mass of active entrants.¹⁸ We then have the following result (see Appendix A.1.3).

Proposition 3 *When entrants invest in horizontal innovation only, their investment in innovation satisfies*

$$h(\eta) = \sqrt{\frac{2(r\kappa + \eta\rho_W\sigma_W)}{\zeta_h}}, \quad (33)$$

which is an increasing function of η .

The incentive to invest in horizontal innovation stems from the perspective of becoming an exploiter. Thus, if the exploiter value increases with η , the optimal horizontal innovation

¹⁸Recall that a vertical breakthrough turns an entrant into the new incumbent initiator, whereas a horizontal innovation turns an entrant into an exploiter.

rate h increases too.¹⁹ This property is noteworthy because, as we show in the analysis of the full model featuring both vertical and horizontal innovation (see Section 3.2.3 below), this monotonicity does not hold. That is, the analysis in these corner cases helps us pin down the strengths at play in the full model, which we analyze next.

3.2.3 The full model

When entrants invest in both horizontal and vertical innovation, the rate of creative destruction and the rate of horizontal displacement are solved endogenously through the maximization problem of the entrants, which depends on the prospect of becoming an initiator or an exploiter. In turn, the values of the initiator and the exploiters depend in equilibrium on the rate of creative destruction and the rate of horizontal displacement. While the richness of our model prevents us from deriving these quantities analytically, we investigate the predictions of our model numerically.

Baseline parameterization Table 1 reports our baseline parameterization. We set the risk-free rate to 1%. Following previous models of innovation, we normalize $\phi = \phi_v = \phi_h$ to one.²⁰ We assume that the R&D cost parameter for entrants’ vertical innovations ζ_v (i.e., for innovations that improve a technology that the firm does not currently own) is ten times larger than for the initiator’s innovation ζ (i.e., for innovations improving a technology the firm already has expertise about), which is in the ballpark of [Akcigit and Kerr \(2018\)](#). Furthermore, we assume that ζ_h is smaller than ζ_v to acknowledge that horizontal innovation—being more exploitative than explorative—is less costly than vertical innovation. The values of the quality jumps upon a breakthrough $\lambda = 1.055$ and $\Lambda = 1.12$ are also in line with [Akcigit and Kerr \(2018\)](#). We set $\varphi = 1.14$, which is consistent with the estimates of [Argente, Lee, and Moreira \(2020\)](#) about the contribution of new products to sales growth. Notably, the inequality $\varphi > \lambda$ implies that the breakthroughs by the

¹⁹In turn, to solve for μ and z , we need to solve the system of equations including the HJB of the initiator and of the exploiters (see Appendix A.1.3).

²⁰See, for instance, [Akcigit and Kerr \(2018\)](#) or [Akcigit, Hanley, and Serrano-Velarde \(2021\)](#), who also study heterogeneous innovations.

initiator is more exploitative than explorative, consistent with [Gao, Hsu, and Li \(2018\)](#) among others.²¹ We set δ to 0.2, which captures the overlap between existing and new innovations reported by the [OECD \(2015\)](#).²² Furthermore, we set ω to 0.25, which implies that horizontal innovations lead to a 5% drop in the initiator’s output, which is consistent with the estimates of [Kogan et al. \(2017\)](#).

We set $\beta = 0.13$, so that the markup is consistent with the estimates by [Hall \(2018\)](#). Moreover, in our one-state model, we normalize $\Gamma = 1$. We calibrate σ so that the cash flow volatility of the initiator is about 11% (as in [Malamud and Zucchi, 2019](#)). Moreover, we assume that $\sigma_X < \sigma$ to acknowledge that, differently from initiators, exploiters do not have an active R&D program and, thus, their cash flow volatility is smaller.²³ In turn, we assume that the entrants’ volatility is greater and equal to 20%, which is consistent with [Begenau and Palazzo \(2021\)](#) who show that entrants have increasingly exhibited greater volatility and R&D expenditures over time. Moreover, we acknowledge that entrants, as they are exposed to the technological risk of their R&D ventures, are comparatively more exposed to idiosyncratic risk than actively-producing firms (initiator and exploiter)—consistently, we assume that $\rho = 0.55$ and $\rho_W = 0.2$. We set the recovery rate in liquidation of the exploiters—which do not invest in innovation—to 0.85, consistent with [Korteweg \(2010\)](#). By setting a lower recovery rate for the initiator—which invests in innovation—we recognize that R&D entails asset intangibility, which leads to a greater value loss in liquidation. We set the magnitude of the entry cost to $\kappa_E = 0.015$, which gives a rate of creative destruction consistent with [Acemoglu et al. \(2018\)](#).

The equilibrium impact of the market price of risk on innovation We start by investigating the sensitivity of the model’s endogenous quantities to the market price of risk η . Consistent with the analytical results in [Proposition 2](#), [Figure 1](#) shows that z and v increase with η in the full model. That is, when considering the endogenous industry

²¹Consistently, [Arora, Belenzon, and Sheer \(2021\)](#) find that firms are investing more in development in recent years.

²²The degree of overlap is captured by the backward citation index, see [OECD \(2015\)](#). The report shows that, depending on the sector, the index ranges from slightly below 0.1 to slightly above 0.3.

²³Recall that volatility for these firms is given by σ_X and $\sigma_X X_s$.

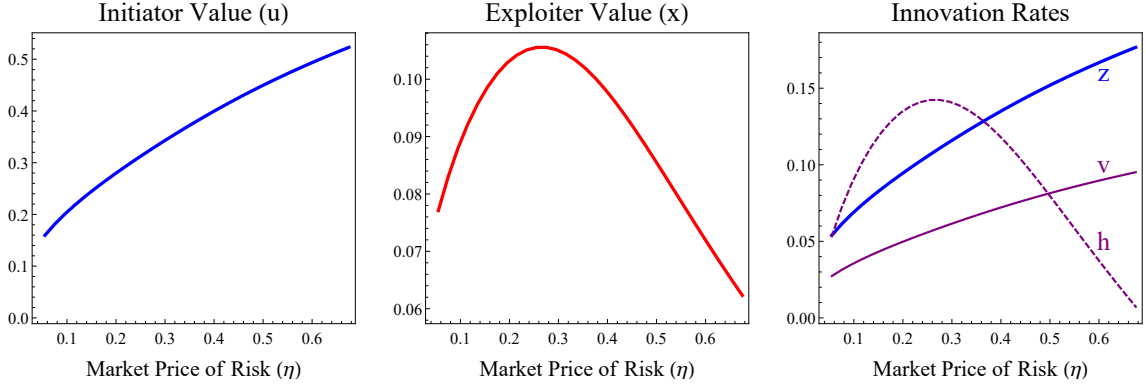


Figure 1: FIRM VALUES AND OPTIMAL INNOVATION RATES. The figure shows the initiator’s and the exploiters values, as well as the initiator’s and the entrants’ optimal innovation rates (both vertical and horizontal) as a function of η .

equilibrium—and, thus, recognizing that firm’s incentives to invest in innovation depend on the industry structure in which a firm operates—the market price of risk has a positive effect on active firms’ innovation rate aimed at starting new technological clusters (i.e., the more explorative type of innovation). Notably, this result overturns the conventional wisdom (supported by Proposition 1 in which we abstract from industry dynamics) that discount rates frustrate long-term risky investment such as R&D.

Whereas a higher market price of risk supports the vertical innovation rate of active entrants (the intensive margin), Figure 2 indicates that it has a negative impact on the mass of active entrants (the extensive margin), which is shown to be decreasing with η . That is, our model suggests that a higher η effectively acts as an entry barrier. Confirming the result in Proposition 2, Figure 2 shows that the declining pattern of μ more than offsets the increasing pattern of v in η —as a result, the rate of creative destruction Ψ_v decreases with η . A lower Ψ_v implies less competitive pressure on active firms, which spurs the more explorative R&D investment aimed at starting new technological clusters.

Moving to horizontal innovation, Figure 1 shows that h is hump-shaped in η . As illustrated by equation (22), the entrants’ investment in horizontal innovation is directly linked to the prospect of becoming an exploiter. Consistently, Figure 1 shows that the sensitivity of h to η is largely driven by the non-monotonic impact of η on the exploiter

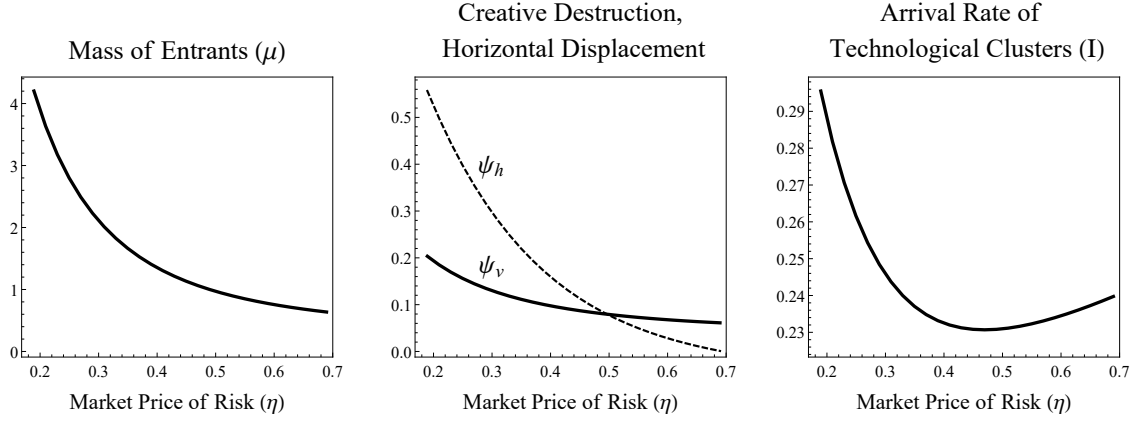


Figure 2: INDUSTRY EQUILIBRIUM AND THE MARKET PRICE OF RISK. The figure shows the mass of active entrants (μ), the rate of creative destruction (Ψ_v), the rate of horizontal displacement (Ψ_h), and the endogenous arrival rate of new technological clusters (\mathcal{I}) as a function of η .

value, x .²⁴ This result is in contrast with Proposition 3, showing that h increases with η if entrants only invest in horizontal innovation. In fact, the interaction between competition in the vertical and horizontal dimensions triggers nontrivial dynamics. At lower levels of η , h increases with η as the rate of creative destruction concurrently declines, and so does the associated liquidation risk of the exploiter—i.e., the lower threat of creative destruction spurs horizontal innovation. However, as η increases further, the higher innovation rate of the initiator increases the exploiters’ liquidation risk, leads to a decline in the exploiter value x , and reduces the entrants’ incentives to invest in horizontal innovation.

These results illustrate that a higher market price of risk stimulates the explorative type of innovation and deters the exploitative type. This can also be seen at the industry level, through the relative magnitude of Ψ_h and Ψ_v (see Figure 2, middle panel). When η is sufficiently low, the rate of horizontal displacement Ψ_h is greater than the rate of creative destruction Ψ_v , meaning that entrants as a whole invest more in horizontal innovation—thus, innovation is more exploitative. Conversely, when η is sufficiently high, the rate of creative destruction Ψ_v is greater than the rate of horizontal displacement Ψ_h , meaning that entrants invest more in vertical innovation—thus, innovation is more explorative.

²⁴Conversely, the figure shows that the initiator value increases with η , consistent with Proposition 2.

A question then arises as to what is the net impact of the market price of risk on the rate \mathcal{I} at which new technological clusters arise. As illustrated by equation (25), this quantity is the sum of the contribution of the initiator (ϕz) and of the entrants (Ψ_v). Because z increases whereas Ψ_v decreases with η , the sensitivity of \mathcal{I} to η is ambiguous. Figure 2 shows that, under our baseline parameterization, \mathcal{I} is U-shaped in η . That is, perhaps surprisingly, our model shows that an increase in the market price of risk can stimulate the advent of new technological clusters. Again, this prediction contrasts with the textbook intuition that discount rates frustrate long-term investment. Our model suggests that the market price of risk importantly affects the composition of innovation within an industry—hence, a higher market price of risk need not lead to a reduction in the industry innovation rate.

The interaction between vertical and horizontal innovation As illustrated above, considering both vertical and horizontal R&D is key to understand the equilibrium dynamics of an innovative industry and its sensitivity to the market price of risk. To further investigate the interaction between vertical and horizontal innovation, Table 2 exhibits the model’s endogenous quantities when considering the cases in which entrants invest in horizontal or vertical innovation only (as analyzed in Section 3.2.2) and in the full case, for different values of ω . Because horizontal innovation is less appealing when ω is smaller—as horizontal breakthroughs create a smaller mass of new products and, thus, are less profitable—the case with horizontal innovation only exists *if* ω is sufficiently large (i.e., the bottom panel of the table).

Intuitively, introducing horizontal innovation—i.e., moving from the case with vertical innovation only to the full case—has an ambiguous effect on the mass of active entrants μ . If horizontal innovation is sufficiently appealing (ω is larger), μ should increase when moving to the full case. At the same time, however, horizontal innovation frustrates vertical innovation by making the initiator and exploiters more exposed to product obsolescence—a strength that reduces the mass of entrants μ . Table 2 suggests that, under our baseline parameterization, the second strength dominates and, thus, μ decreases when introducing

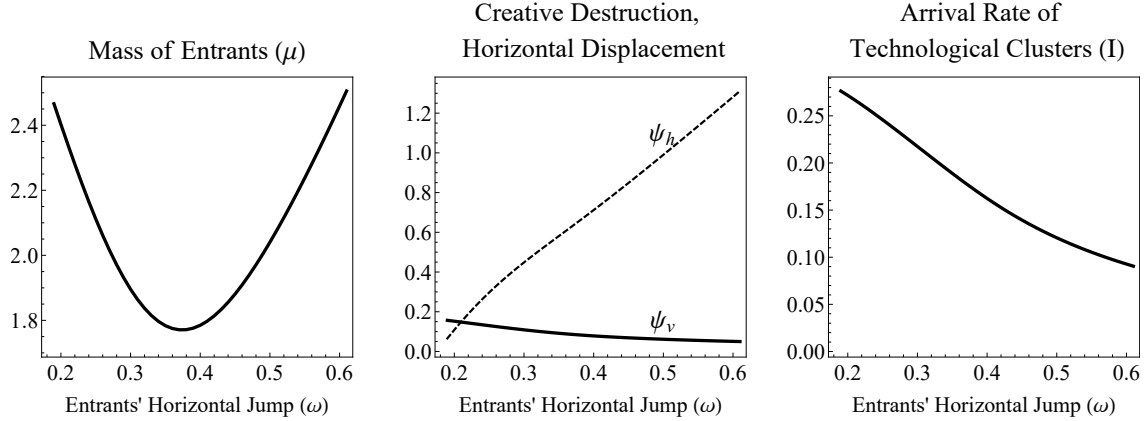


Figure 3: THE IMPACT OF HORIZONTAL INNOVATION ON THE INDUSTRY EQUILIBRIUM. The figure shows the mass of active entrants μ , the rate of creative destruction (Ψ_v) and of horizontal displacement (Ψ_h), and the endogenous arrival rate of technological clusters (\mathcal{I}) as a function of ω .

horizontal innovation.²⁵

In the full case with horizontal and vertical innovation, these opposing strengths imply that there is a tension regarding the effects of horizontal innovation (gauged by the parameter ω) on entry incentives. Indeed, Figure 3 illustrates that the mass of active entrants μ is U-shaped in ω . The reason is that the surplus from horizontal innovation increases with ω , but the surplus from vertical innovation simultaneously declines. As a result, the rate of horizontal displacement Ψ_h sharply increases, whereas the rate of creative destruction Ψ_v decreases with ω , which causes the arrival rate of new technological clusters \mathcal{I} to be decreasing in ω too.

Table 2 also shows that horizontal innovation decreases the initiator’s innovation (z). The drop is wider if ω is larger, in which case horizontal breakthroughs trigger a sharper drop in the initiator’s product lines. Similarly, the entrants’ rate of vertical innovation v drops notably if ω is larger. Folding in the effects on μ and v , Table 2 shows that the rate of creative destruction decreases when introducing horizontal innovation—i.e., Ψ_v is lower in the full case than in the case with vertical innovation only. This result, together

²⁵In unreported results, we find that the first strength dominates if ω is unrealistically high, in which case the incentives to invest in horizontal innovation are disproportionately greater than the incentives to invest in vertical innovation.

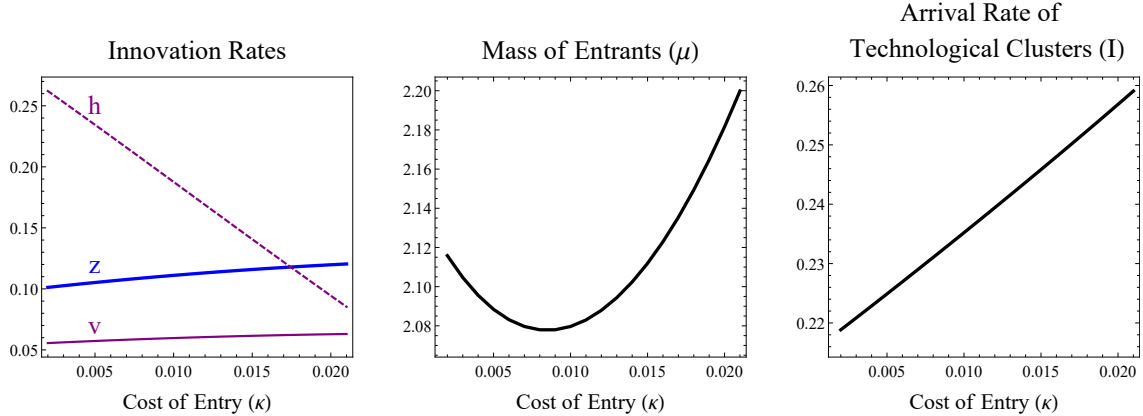


Figure 4: COST OF ENTRY AND THE INDUSTRY EQUILIBRIUM. The figure shows the initiator’s and the entrants’ optimal innovation rates (z , v , and h), the mass of active entrants (μ), and the endogenous arrival rate of technological clusters (\mathcal{I}) as a function of κ .

with the aforementioned impact on z , implies that horizontal innovation frustrates vertical innovation. It also implies that a greater emphasis on horizontal innovation leads to a lower industry turnover—i.e., on average, the initiator is expected to remain the technology leader for longer.

Consider now the effect of introducing vertical innovation on the industry equilibrium, i.e., moving from the case with horizontal innovation only to the full case. Table 2 shows that vertical innovation has a positive impact on the rate of horizontal displacement. This result is consistent with the evidence in Braguinsky et al. (2020), showing that vertical innovations have notable spillovers into horizontal expansions. In fact, the upside associated with vertical innovation—i.e., the prospect of becoming the next industry incumbent—spurs an increase in the mass of active entrants that, in turn, boosts the aggregate rate of horizontal displacement Ψ_h too. This increase is driven by the sharp rise in the mass of active entrants. At the same time, the rate of horizontal innovation h sharply drops when moving to the full case—i.e., because vertical innovation promises a greater upside potential (the perspective of becoming the new industry initiator), active entrants in the full case shift from horizontal to vertical innovation.

To further understand the interaction between vertical and horizontal innovation, Figure

4 investigates how the entrants' cost of entry—as gauged by the parameter κ —affects R&D investment. Our model shows that the magnitude of the entry cost largely impacts the type of innovation pursued by firms in the industry. Namely, a greater cost of entry fosters vertical innovation but deters horizontal innovation. As shown by Figure 4, this translates into the mass of active entrants being U-shaped with respect to κ . In fact, for low levels of κ , a greater cost of entry curbs entrants' incentives to invest in horizontal innovation; by contrast, when κ is sufficiently large, the negative effect of κ on horizontal innovation by entrants is shadowed by their increased incentive to innovate vertically. This positive effect, together with the increasing pattern of z in κ , jointly explain why the arrival rate of technological clusters increases with the cost of entry.

4 Time-varying market price of risk

We now assume that the market price of risk varies with the state of the economy, being η_G in the good state and $\eta_B > \eta_G$ in the bad state. For the sake of brevity, we report the derivation of the firm's optimal choices and of the industry equilibrium in Appendix A.2.

Before analyzing the full model with both vertical and horizontal innovation, it is worth considering again the corner cases analogous to those in Section 3.2.2 for the one-state model. We show the following results (proofs are in Appendix A.2.2).

Proposition 4 *Consider the case in which the market price of risk is time varying and the inequality $\eta_B > \eta_G$ holds. If entrants only engage in vertical innovation, the initiator's and active entrants' innovation rates satisfy, respectively,*

$$z_j(\eta_j, \eta_{j-}) = \frac{\phi}{\zeta} (\lambda\varphi - 1) u_j(\eta_j, \eta_{j-}),$$

and

$$v_j(\eta_j, \eta_{j-}) = \frac{\phi_v}{\zeta_v} [\Lambda u_j(\eta_j, \eta_{j-}) - \kappa],$$

and are countercyclical—i.e., $z_B > z_G$ and $v_B > v_G$. Conversely, the rate of creative

destruction Ψ_{vj} and the mass of active entrants μ_j are procyclical—i.e., $\Psi_{vG} > \Psi_{vB}$ and $\mu_G > \mu_B$. The entrants' extensive innovation margin (μ_j) is more sensitive to variations in the market price of risk than the intensive margin (v_j). If, instead, the entrants only engage in horizontal innovation, their optimal innovation rate satisfies

$$h_j(\eta_j, \eta_{j-}) = \frac{\phi_h}{\zeta_h} [\omega x_j(\eta_j, \eta_{j-}) - \kappa]. \quad (34)$$

and is countercyclical—i.e., $h_B > h_G$.

Proposition 4 illustrates how time variation in the market price of risk affects the industry equilibrium. The proposition suggests that the innovation rate of active firms is countercyclical—i.e., it is greater when the market price of risk is larger (in the bad state). This holds in both corner cases with either vertical or horizontal innovation only. Proposition 4 also shows that the greater market price of risk in state B bears a negative impact on the *extensive* margin—i.e., the mass of active entrants declines. Moreover, with time-varying discount rates, the variation in the extensive margin is greater than the corresponding variation in the intensive margin. This implies that the procyclicality of μ_i then extends to the rate of creative destruction, which is procyclical too. That is, in the good (bad) state, the mass of active entrants is larger (smaller), creative destruction is the highest (lowest), and incumbent firms reduce (increase) their R&D investment.

As we show in our quantitative analysis that follows, the greater sensitivity of the extensive margin to variations in the market price of risk has important implications for the average impact of fluctuations on industry outcomes vis-à-vis an identical economy featuring no fluctuations.

Understanding innovation cyclicity through variations in the market price of risk We next analyze the model featuring both vertical and horizontal innovation. On top of the parameters in Table 1, we assume that $\tilde{\pi}_G = 0.1$ and $\tilde{\pi}_B = 0.4$ under the physical measure, meaning that the good and the bad states are expected to last 10 years and 2.5 years, respectively. Moreover, we set $\theta_G = -\theta_B = 0.08$, which implies that risk averse

investors expect the good state to be shorter and the bad state to last longer than under the physical measure. Throughout the analysis, we consider two cases. First, we only allow η_i to vary across different states and assume that $\Gamma_G = \Gamma_B = 1$ —i.e., the demand shift parameter is constant. Second, to acknowledge that variations in demand are an important component of firms’ innovation incentives since Schumpeter (see also Caballero and Hammour, 1994), we allow the demand function to vary as illustrated in equation (2)—namely, we keep $\Gamma_B = 1$ and assume that $\Gamma_G = 1.02$, so that the wedge of industry profits between the good and the bad state is above 30% in our baseline.²⁶

Table 3 compares the endogenous quantities in the G and B states. In the top panel, we only vary η_i across the states, whereas we also vary the demand shift Γ_i in the bottom panel. Consistent with the result in Proposition 4, we find that investment in innovation by active firms is countercyclical—i.e., it is higher in state B , in which the market price of risk is larger. This result is in line with the Schumpeterian view that firms should invest more in innovation in recessions than in expansion, as the opportunity cost of foregone revenues is smaller. Importantly, in our model, this is the case both when abstracting (top panel) and when accounting for time-varying demand (bottom panel)—i.e., fluctuations in the market price of risk can, by themselves, generate this pattern.

Furthermore, Table 3 shows that the mass of active entrants is procyclical—i.e., fewer entrants are active when the market price of risk is larger in the B state. That is, the higher market price of risk in the B state has the most detrimental effect on the extensive margin, by reducing the mass of firms actively investing in innovation. At the industry level, the table also shows that the procyclicality of μ more than offsets the countercyclicality of firm-level innovation in both the vertical and horizontal dimension—as a result, Ψ_v and Ψ_h are both procyclical. That is, active firms face a greater competition in the innovation dimension—through greater creative destruction and a greater rate of horizontal innovation—when the market price of risk is lower in the good state. The analysis also

²⁶The variation in profits across states in our model is therefore consistent with the change we observe between peaks and troughs in total (detrended) earnings before interest, depreciation and amortization of R&D-active firms in Compustat.

shows that the aggregate innovation rate at which new technological clusters arise, \mathcal{I} , is procyclical too.

Our analysis then illustrates that variations in discount rates are an important driver of innovation cyclicity within an industry. Namely, when the market price of risk is high (in bad states of the economy), the mass of entrants should shrink, and active firms (initiator and entrants) should invest more in innovation. In contrast, when the market price of risk is low (in good states of the economy), we should see a considerable increase in the mass of entrants, which in turn should reduce the innovation rate of active firms. In other words, these results provide novel theoretical grounds to the empirical literature that investigates the cyclical behavior of R&D investment, by posing the accent on a relatively unexplored aspect, i.e., the effect of fluctuations in discount rates on an industry’s R&D composition.

Our paper can thus rationalize the observed procyclical innovation rates at the aggregate level—a pattern that has been consistently reported starting from [Griliches \(1984\)](#)—which appear to be at odds with the Schumpeterian prediction that firms should invest more in recession than in expansion. Consistent with the empirical work by [Howell et al. \(2020\)](#), we show that strengths steering pro- and countercyclicity coexist. Indeed, our model shows that the lack of countercyclicity comes mostly from the extensive margin, a pattern that is in line with the evidence in [Babina, Bernstein, and Mezzanotti \(2020\)](#).²⁷ Also consistent with [Babina, Bernstein, and Mezzanotti \(2020\)](#), we show that incumbent firms’ R&D engagement does not dwindle in bad states of the economy. Furthermore, in line with [Howell et al. \(2020\)](#), our paper shows that the aggregate contribution of entrants to innovation is higher in good states of the economy—this notwithstanding, the firm-level investment of active startups is higher in bad states, as predicted by our model.

The impact of fluctuations in the market price of risk Having analyzed how the varying level of the market price of risk affects R&D cyclicity, we next investigate the effect of these fluctuations vis-à-vis an environment in which η is fixed. To this end, the

²⁷This pattern is also consistent with [Brown, Fazzari, and Petersen \(2009\)](#), who focus on financing frictions as an important contributor.

last two columns of Table 3 report the endogenous quantities of interest in the one-state model (as analyzed in Section 3) as well as their averages in the two-state case. To make the one- and the two-state cases comparable, we assume that the time-invariant market price of risk in the one-state is equal to the average in the two-state model.²⁸

Table 3 shows that fluctuations in the market price of risk affect the firm-level innovation rate only slightly under our baseline parameterization, with the entrants' horizontal innovation rate being only modestly smaller in the two-state model on average. In turn, fluctuations in the market price of risk have a considerable impact on the average mass of active entrants, consistent with Proposition 4. Table 3 shows that the average mass of active entrants in the two-state model is greater than its counterpart in the one-state. Moreover, the greater mass of active entrants implies that the rate of creative destruction is greater, on average, in the two-states vis-à-vis the one-state case. As a result, the rate of arrival of new technological clusters \mathcal{I} is greater in the presence of fluctuations in the market price of risk, on average. In other words, our model shows that these fluctuations induce a greater industry turnover which, in turn, is beneficial to the emergence of new technological clusters. These patterns are confirmed in the bottom panel, in which we also account for demand-shifts over the business cycle. That is, our paper supports the view that fluctuations are not detrimental to the industry equilibrium.²⁹

We also investigate the sensitivity of the equilibrium quantities with respect to the market price of risk in the two states, η_G and η_B . Figure 5 shows that z_j increases with the magnitude of the market price of risk in the contemporaneous state η_j —consistent with the result in the one-state model—whereas it is quite insensitive to the market price of risk in the non-contemporaneous state η_{j^-} . By contrast, the mass of active entrants μ_j and the arrival rate of new technological clusters \mathcal{I}_j are notably sensitive to the market price of risk in both the contemporaneous and in the non-contemporaneous states, j and j^- . Namely,

²⁸I.e., we assume that $\bar{\eta} = \frac{\eta_G\pi_B + \eta_B\pi_G}{\pi_B + \pi_G}$ —i.e., in the one-state. Similarly, in the bottom panel, we set $\bar{\Gamma} = \frac{\Gamma_G\pi_B + \Gamma_B\pi_G}{\pi_B + \pi_G}$ in the one-state.

²⁹Manso, Balsmeier, and Fleming (2019) show that macroeconomic fluctuations stir a more balanced mix between explorative and exploitative innovation. In line with Manso, Balsmeier, and Fleming (2019), in unreported results we find that h can be procyclical when allowing for time variation in Γ .

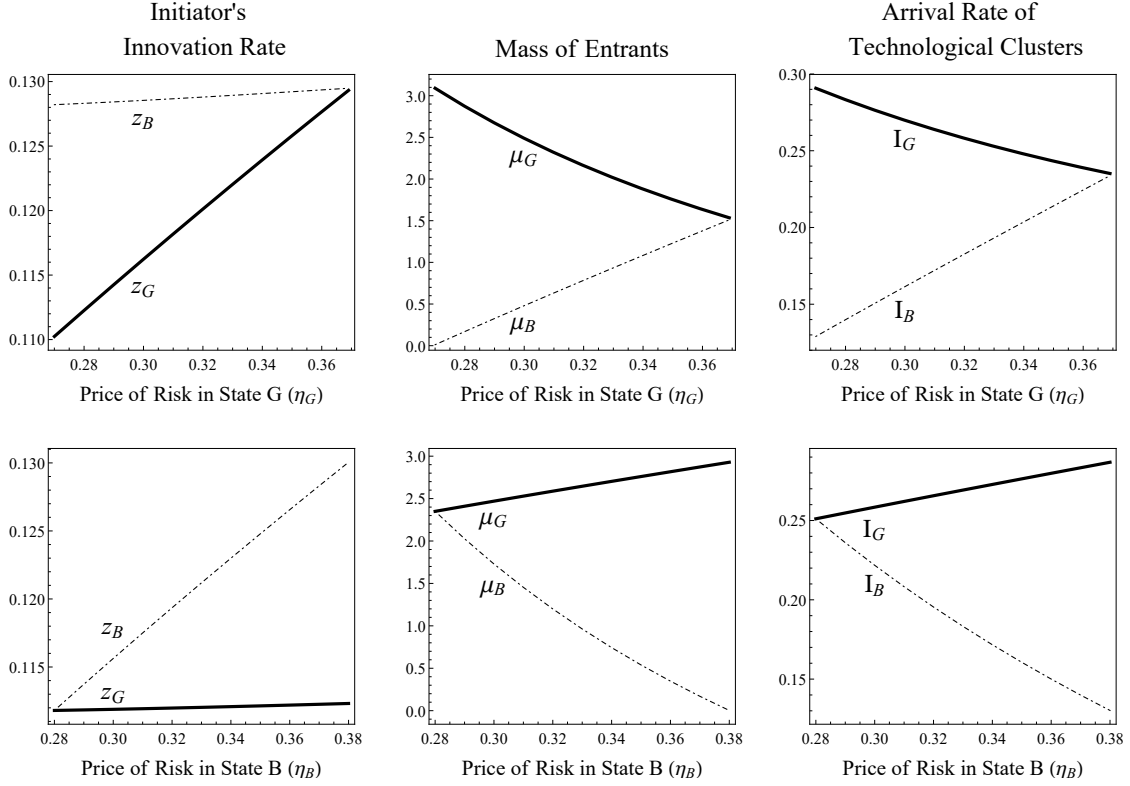


Figure 5: TIME-VARYING EXTENSIVE AND INTENSIVE MARGINS. The figure shows how the initiator's innovation rate (z_j), the mass of entrants (μ_j), and the endogenous arrival rate of technological clusters (\mathcal{I}_j) vary with the market price of risk either in the contemporaneous state (η_j) or in the non-contemporaneous state (η_{-j} , bottom panel). The top panel varies the market price of risk in the G state, whereas the bottom panel varies the market price of risk in the B state.

these quantities decrease with the market price of risk in the contemporaneous state j , but increase with the market price of risk associated with the non-contemporaneous state j^- . Specifically, an increase in the market price of risk shifts entrants from the contemporaneous to the non-contemporaneous state, then also affecting the arrival rate of new technologies. This effect then sheds light on the importance of the market price of risk in transferring resources across states of the economy.

5 Asset Pricing Implications

We conclude our analysis by studying the asset pricing implications of our model. An important insight that stems from our analysis is that the resolution of the idiosyncratic uncertainty associated with innovation can dramatically affect firms' risk premia (Berk, Green, and Naik, 2004). Each firm in our model is subject to the two sources of systematic risk: the diffusion risk and the jump risk associated to discrete changes in the state of the economy. Simultaneously, firms are subject to different sources of idiosyncratic risk: the idiosyncratic component of cash flow risk, uncertainty associated to its own innovation outcomes, or shocks linked to vertical or horizontal innovation by rival firms (i.e., creative destruction or horizontal displacement). Although there is no risk premium earned on idiosyncratic risk per se, the expected resolution of innovation uncertainty either augments or mitigates firms' exposure to systematic risk. Firms' innovation strategies vary with systematic changes in discount rates, and risk premia adjust accordingly.

A heuristic derivation of risk premia in the two state model involves a comparison of the HJB equations under the physical and risk-neutral measures, as in Bolton, Chen, and Wang (2013). We define risk premia as expected returns in excess of the risk free rate r . Let the risk premium of the initiator in state j be $\mathcal{R}_{U,j}$, and that of the exploiter be $\mathcal{R}_{X,j}$.³⁰ For the initiator, we obtain the expression:

$$\mathcal{R}_{U,j} \equiv \rho\sigma\eta_j \frac{y_j}{u_j} + \tilde{\pi}_j (e^{\theta_j} - 1) \frac{u_j - u_{-j}}{u_j}, \quad (35)$$

where the first term is the risk premium associated with the diffusion risk, whereas the second term is the premium associated with the jump risk.³¹ Similarly, the expression for the risk premium of the exploiter is given by:

$$\mathcal{R}_{X,j} \equiv \rho\sigma_X\eta_j \frac{\omega}{1 - \delta\omega} \frac{y_{X,j}}{x_j} + \tilde{\pi}_j (e^{\theta_j} - 1) \frac{x_j - x_{-j}}{x_j}. \quad (36)$$

³⁰We do not analyze the corresponding risk premium of the entrant as it is likely unobservable by the econometrician (entrants should be interpreted as startups): It corresponds to a risk premium on seed capital equal to $\rho_W\sigma_W \frac{\eta_j}{\kappa}$.

³¹The risk premium in the one-state case can be thus obtained by setting $\pi_j = 0$ in equation (35).

Competitive pressure by entrants and risk premia The first core prediction of the model refers to how *competitive pressure* by entrants affects incumbents' (initiator and exploiters) risk premia. To disentangle the strengths at play within the industry, we start by considering these risk premia in the corner cases in which entrants engage exclusively in either vertical or horizontal innovation, for which we obtain analytical solutions.

Proposition 5 *If entrants only engage in vertical innovation, the initiator's risk premium, $\mathcal{R}_{U,j}$, is increasing in both the frequency and size of entrants' innovations (ϕ_v and Λ). Similarly, if the entrants only engage in horizontal innovation, the exploiter's risk premium, $\mathcal{R}_{X,j}$, is increasing in the frequency and size of entrants' innovations (ϕ_h and ω) and in the degree of overlap between ensuing new products and those of the initiator (δ).*

Proposition 5 shows that the key parameters that affect the entrants' innovation rate impact the risk premia of the initiator and of the exploiter (proofs are in Appendix A.3.1). Namely, if entrants engage exclusively in vertical innovation, an increase in the likelihood of a technological breakthrough by entrants (i.e., an increase in ϕ_v) results in higher risk premia for the initiator. Similarly, when the scale of breakthroughs is greater (captured by a higher value of Λ), the entrants' investment on vertical innovation v increases, the competition by entrants also increases through an increase in the rate of creative destruction, and hence there is a greater likelihood of liquidation for the initiator. It follows that greater *competitive pressure* by entrants makes the initiator riskier.

Similarly, if entrants engage exclusively in horizontal innovation, Proposition 5 indicates that the prospect of greater displacement in product lines increases the risk premia of exploiters. A rise in the likelihood of horizontal innovation by entrants (i.e., an increase in ϕ_h), or more significant horizontal displacement by entrants (captured by a higher value of ω or δ), consistently result in higher risk premia for the exploiters.³²

The qualitative predictions in Proposition 5 are confirmed in the full model with both vertical and horizontal innovation. Moreover, in the full case, the risk premium of the

³²As we cannot solve for the initiator's risk premium in closed form in the case with horizontal innovation only, our analytical results focus on the exploiters' risk premium.

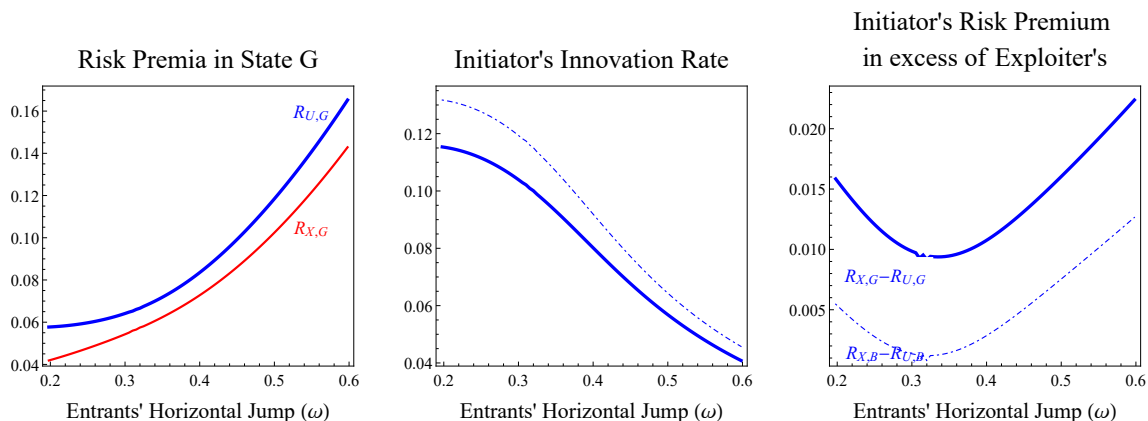


Figure 6: RISK PREMIA AND HORIZONTAL DISPLACEMENT. The figure shows the risk premia of the initiator and of the exploiters in state G , the initiator’s optimal innovation rate, and the risk premium of the initiator in excess of the exploiters’ as a function of entrants’ horizontal jump ω in the full model featuring both vertical and horizontal innovation.

initiator is also affected by the obsolescence triggered by horizontal innovation: Figure 6 confirms that the risk premium of the initiator is increasing in ω . Intuitively, a higher ω boosts the incentives of entrants to innovate horizontally, and in turn erodes the innovation incentives of the initiator, whose risk premium then increases with ω in either state.³³ Figure 6 additionally shows that the risk premium of the initiator in excess of that of exploiters is U-shaped in ω . Thus, while an increase in horizontal displacement increases the risk premia of both initiator and exploiters in the full model, such increase may affect mostly either the initiator or exploiters, in any state.

Put together, our findings in Proposition 5 and in the full model thus suggest that innovation by competing entrants make incumbents (either initiator or exploiters) *riskier*—regardless of whether innovation by entrants is vertical or horizontal. By contrast, the literature on product market competition suggests that endogenous entry by rival firms makes incumbents *safer* (i.e., [Bustamante and Donangelo, 2017](#); [Babenko, Tserlukevich, and Boguth, 2020](#)). In these models, unlike ours, firms compete for market share in the same product market: Firms do not innovate nor compete in innovation, and entry by new firms

³³The pattern on incumbents’ risk premia with respect to ω for state B is qualitatively the same as that shown in Figure for state G .

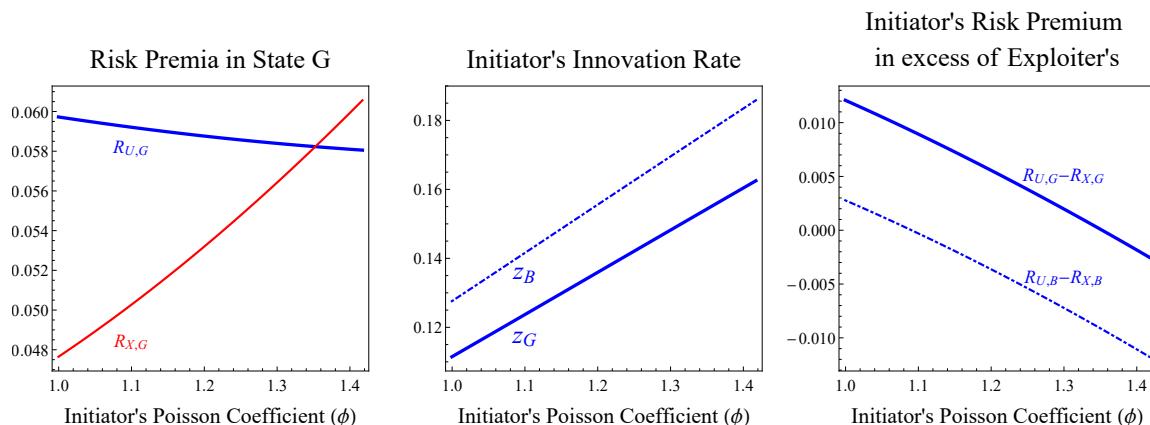


Figure 7: RISK PREMIA AND THE INITIATOR’S INNOVATION INTENSITY. The figure shows the risk premia of the initiator and of the exploiters in state G, the initiator’s optimal innovation rate, and the risk premium of the initiator in excess of the exploiters’ as a function of the initiator’s Poisson intensity ϕ in the full model featuring both vertical and horizontal innovation.

erodes firms’ markups in equilibrium. It follows that the source of rivalry among competing firms—i.e., whether firms compete in innovation or for market share in product markets—is critical to understand how actions by entrants affect the risk premia of incumbents. Our model adds to the insights in [Bloom, Schankerman, and Van Reenen \(2013\)](#) in providing a novel mechanism to identify the nature of firm rivalry, through the impact of rival entry on the expected returns of incumbents.

Initiator-exploiter interactions and risk premia The full model also characterizes how the interactions between the initiator and exploiters affect risk premia. In particular, we find that the risk premium of the initiator decreases if its own investment in vertical innovation increases—indicating that the ability of the initiator to innovate acts literally as insurance against competition in innovation. In our calibration, a higher intensity of technological breakthrough by the incumbent (captured by an increase in ϕ) boosts the incentives of the initiator to invest in innovation (leading to a higher value of z_j), which in turn reduces its risk premium. Simultaneously, an increase in ϕ leads to a higher risk premium of the exploiter, as it increases its liquidation probability associated with an initiator’s breakthrough. Figure 7 illustrates these findings, and shows that the difference in

risk premia across incumbents, $\mathcal{R}_{U,j} - \mathcal{R}_{X,j}$ decreases for higher values of ϕ .

In sum, either when we look at interactions between entrants and incumbents or between the initiator and exploiters, the model yields the same qualitative prediction. Innovation by competitors increases a firm’s risk premium, whereas a firm’s own innovation acts as insurance against innovation by rival firms. These qualitative results are consistent with the patent race model of two firms by [Bena and Garlappi \(2020\)](#), in which the expected return of one firm decreases with its own innovation output and increases with that of its rival. [Bena and Garlappi \(2020\)](#), however, also predict that the leader in the patent race is always safer. Our model with heterogeneous innovation and endogenous entry suggests that the initiator needs not be safer than the exploiters in equilibrium, as indeed shown in the right panel of [Figure 7](#). Thus, the ability of the initiator to innovate and hence compete in the technological dimension—vis-à-vis the lack of innovation by the exploiters—is not sufficient to make the exploiters riskier unconditionally.

6 Concluding remarks

The study of corporate innovation—and in particular, of its underlying determinants—is key to understand the real economy. Our paper highlights that discount rates are an important determinant of R&D investment decisions. In contrast with the conventional wisdom that higher discount rates deter long-term investment, we show that higher discount rates can encourage innovation in the intensive margin, and spur the emergence of new technologies stemming from explorative innovation.

Our results further highlight that studying the impact of discount rates when allowing for competition in innovation—that is, firms’ rivalry in the technology space—is key to understand the composition and nature of R&D at the industry level. The model reconciles the Schumpeterian view that firms should innovate more intensively in recessions, with the observed procyclicality of aggregate R&D investment. Allowing for firms’ interaction in the technology space is also key to understand the interrelation of firms’ risk premia.

Overall, our findings shed light on the importance of studying firms' innovation decisions in industry equilibrium while accounting for the level and time-variation of discount rates—two determinants of R&D investments that are usually overlooked in macroeconomic studies, and on which we intend to elaborate further in future research.

A Appendix

A.1 Proofs of the results in Section 3

Using Girsanov theorem, the risk-neutral dynamics of entrants' cash flows satisfy

$$dC_t^W = \left[- \left(\frac{1}{2} \zeta_v v_t^2 + \frac{1}{2} \zeta_h h_t^2 + \eta \rho_W \sigma_W \right) dt + \sigma_W dB_t^W \right] M_t q_t \quad (37)$$

where we denote by B_t^W the risk-neutral counterpart of \tilde{B}_t^W . Similarly, the risk-neutral dynamics of the exploiters' cash flows satisfy

$$dC_t^X = Y_{Xt} (p_{Xt} - 1 - \eta \rho \sigma_X) M_{Xt} dt + \sigma_X Y_{Xt} M_{Xt} d\tilde{B}_t^X \quad (38)$$

where we denote by B_t^X the risk-neutral counterpart of \tilde{B}_t^X .

To complement the model solution reported in Section (3.1), consider the valuation equation of entrants. Substituting equations (21) and (22) back into the HJB equation (20) gives:

$$\begin{aligned} rw = & -\eta \rho_W \sigma_W + \frac{\phi_v^2}{2\zeta_v} [\Lambda u - w]^2 + \frac{\phi_h^2}{2\zeta_h} [\omega x - w]^2 + \phi z (\lambda \varphi - 1) (w - \kappa) \\ & + \Psi_v^- (\Lambda - 1) (w - \kappa) - \Psi_h^- \omega (w - \kappa) \end{aligned} \quad (39)$$

Using the free-entry condition $w = \kappa$, the above equation boils down to

$$rw = -\eta \rho_W \sigma_W + \frac{\phi_v^2}{2\zeta_v} [\Lambda u - w]^2 + \frac{\phi_h^2}{2\zeta_h} [\omega x - w]^2 \quad (40)$$

becomes a function of u and x , which are endogenous. In turn, the valuation equation of the initiator u (equation (14)) is a function of Ψ_v and Ψ_h , which are themselves endogenous functions of μ , u , and x . Similarly, the valuation equation of the exploiter x (equation (16)) depends on z , Ψ_v , and Ψ_h . As a result, we solve the system of equations (14), (16), and (40) to get the endogenous quantities μ , u , and x , which in turn we substitute into equations (13), (21), and (22) to get the optimal innovation rates z , v , and h . Finally, using the expressions for v and h , together with μ , we pin down Ψ_v and Ψ_h .

A.1.1 Proof of Proposition 1

The expression for $z(\eta)$ when considering exogenous industry dynamics simply follows by substituting equation (26)—solved for a given (exogenous) Ψ_v and Ψ_h —into equation (13). Notably, $z(\eta)$ is a function of η through the function $\Upsilon(\eta)$, defined in Section 3.1. As

$$\Upsilon'(\eta) = -\rho \sigma \left(\frac{1 - \beta}{1 + \eta \rho \sigma} \right)^{\frac{1}{\beta}} \Gamma^{\frac{1}{\beta}} < 0$$

Because this quantity decreases with η , then $z(\eta)$ decreases with η too, and the result follows.

A.1.2 Proof of Proposition 2

In the case considered in Proposition 2, entrants only focus on vertical innovation. Thus, the industry features only the initiator and the mass of entrants, and there is no horizontal displacement. In this case, the value of the initiator satisfies the following HJB equation

$$\begin{aligned} rU(q, M) = \max_{z, Y} & MY(p - 1 - \sigma\eta\rho) - \zeta \frac{z^2}{2} qM + \phi z [U(\lambda q, \varphi M) - U(q, M)] \\ & + \Psi_v [\alpha U(q, M) - U(q, M)] \end{aligned} \quad (41)$$

where the expressions for the optimal innovation rate, production quantity, and selling price are given by equations (13) and (12). However, in this equation, Ψ_v is endogenous and derived by solving for the optimal policies of the entrants.

Consider the optimal policies of a given entrant, who only invests in vertical innovation by assumption. The value of an entrant, denoted by $W(q, M)$, satisfies the following HJB equation:

$$\begin{aligned} rW(q, M) = \max_{v \geq 0} & -qM \left(\eta\rho_W\sigma_W + \frac{\zeta_v}{2} v^2 \right) + \phi_v v [U(\Lambda q, M) - W(q, M)] \\ & + \phi z [W(\lambda q, \varphi M) - W(q, M) - K(\lambda\varphi - 1)] + \Psi_v^- [W(\Lambda q, M) - W(q, M) - K(\Lambda - 1)] \end{aligned} \quad (42)$$

where the terms admit a similar interpretation to equation (19). Using the same scaling property used in the full model and differentiating with respect to v gives the expression for the optimal innovation rate reported in equation (21). Plugging the optimal v back into the HJB gives:

$$rw = -\eta\rho_W\sigma_W + \frac{\phi_v^2}{2\zeta_v} [\Lambda u - w]^2 + [\phi z (\varphi\lambda - 1) + \Psi_v^- (\Lambda - 1)] (w - \kappa). \quad (43)$$

Using the free entry condition $w = \kappa$, we can then solve the above equation with respect to u , which then gives equation (29). By substituting u into (13) and (21) then gives the expressions for $z(\eta)$ and $v(\eta)$ reported in Proposition 2. It is straightforward to show that these endogenous quantities increase with η .

We now prove the sensitivity of $\Psi_v(\eta)$ and $\mu(\eta)$ with respect to η . Scaling equation (41) by q and M , substituting the optimal y and z , and solving with respect to $\Psi_v(\eta)$ gives:

$$\begin{aligned} \Psi_v(\eta) &= \frac{1}{(1 - \alpha)u(\eta)} \left[\frac{\phi^2}{2\zeta} (\lambda\varphi - 1)^2 u^2(\eta) - ru(\eta) + \Upsilon(\eta) \right] \\ &= \frac{1}{1 - \alpha} \left(\frac{\phi^2}{2\zeta} u(\eta) (\lambda\varphi - 1)^2 - r + \frac{\Upsilon(\eta)}{u(\eta)} \right) \end{aligned}$$

Differentiating with respect to η gives

$$\Psi_v'(\eta) = \frac{\Upsilon'(\eta)}{u(\eta)(1 - \alpha)} - \left[\Upsilon(\eta) - \frac{\phi^2(\lambda\varphi - 1)^2 u^2(\eta)}{2\zeta} \right] \frac{u'(\eta)}{u^2(\eta)(1 - \alpha)}. \quad (44)$$

The first term is negative as $1 - \alpha > 0$, and $\Upsilon'(\eta) < 0$ as shown in Appendix A.1.1. The second term is also negative, as $u'(\eta) > 0$ (as is straightforward from equation (29)) and the term in parenthesis is positive when we consider values of η that rule out the degenerate case

in which the initiator always makes losses in expectation—i.e., we consider values of η such that $y(p - 1 - \eta\sigma\rho) - \frac{\zeta}{2}z^2 > 0$, as explained in Section 2. Indeed

$$y(p - 1 - \eta\sigma\rho) - \frac{\zeta}{2}z^2 = \Upsilon(\eta) - \frac{\phi^2(\lambda\varphi - 1)^2u^2(\eta)}{2\zeta} > 0$$

is the term in parenthesis in equation (44). Thus, the rate of creative destruction Ψ_v decreases with η , as reported in Proposition 2.

As the last step, we study the monotonicity of $\mu(\eta) = \frac{\Psi_v(\eta)}{\phi_v v(\eta)}$ with respect to η . Differentiating it with respect to η gives

$$\mu'(\eta) = \frac{\Psi'_v(\eta)}{\phi_v v(\eta)} - \frac{\Psi_v(\eta)v'(\eta)}{\phi_v v^2(\eta)} \quad (45)$$

The first term is negative, as $\Psi'_v(\eta) < 0$, as shown above. The second term is also negative, as $\Psi_v(\eta) > 0$ and $v'(\eta) > 0$. The claim in Proposition 2 then follows.

A.1.3 Proof of Proposition 3

In Proposition 3, we assume that entrants only focus on horizontal innovation. In this case, the value of entrants satisfies:

$$\begin{aligned} rW(q, M) = \max_{v, h} & -qM \left(\eta\rho_W\sigma_W + \frac{\zeta_h}{2}h^2 \right) + \phi_h h [X(q, \omega M) - W(q, M)] \\ & + \phi z [W(\lambda q, \varphi M) - W(q, M) - K(\lambda\varphi - 1)] + \Psi_h^- [W(q, M(1 - \omega\delta)) - W(q) + K\omega\delta] \end{aligned} \quad (46)$$

where the terms admit a similar interpretation to equation (19). Using the same scaling property used in the full case and differentiating with respect to h gives the expression for the optimal innovation rate reported in equation (22). Plugging this expression back into the HJB gives and imposing the free-entry condition $w = \kappa$ gives:

$$rw = -\eta\rho_W\sigma_W + \frac{\phi_h^2}{2\zeta_h} [\omega x - w]^2. \quad (47)$$

Solving this equation with respect to x gives equation (32). By substituting x into (22) then gives the expression for $h(\eta)$ reported in Proposition 3.

In this case, differently from the full case, the initiator is not subject to creative destruction Ψ_v . Thus, the value of the initiator satisfies:

$$\frac{\phi^2}{2\zeta} (\lambda\varphi - 1)^2 u^2 - (r + \Psi_h\omega\delta)u + \Upsilon(\eta) = 0. \quad (48)$$

In this case, the exploiters face the threat of exit only due to the initiator's breakthroughs (i.e., they are not subject to creative destruction due to the entrants' innovations). Thus, the value of the exploiters satisfies the following equation:

$$rx = \frac{\beta\omega}{1 - \omega\delta} \left(\frac{1 - \beta}{1 + \sigma_X\rho\eta} \right)^{\frac{1}{\beta} - 1} \Gamma^{\frac{1}{\beta}} - \phi z (1 - \alpha_X) x - \Psi_h\omega\delta x. \quad (49)$$

Now, recall that $\Psi_h = \mu\phi_v h$, where h satisfies the equation reported in Proposition 3. As a result, we can find v and μ by solving the system of equations (48)-(49) and, thus, the optimal innovation rate of initiators z as well as the rate of horizontal displacement Ψ_h .

A.2 Proof of the results in Section 4

A.2.1 Derivation of firm values and optimal investment rates

In the two-state model, all value functions and the endogenous quantities are a function of (η_j, η_{j-}) —i.e., of the market risk prices in the two states. For the ease of exposition throughout this appendix, we omit these arguments.

Consider first the value of the initiator. Following standard arguments, the initiator's scaled HJB equation in each state j satisfies:

$$ru_j = \max_{z_j, y_j} \Gamma_j y_j^{1-\beta} - y_j - \frac{z_j^2}{2} \zeta - \sigma \eta_j \rho y_j + \phi z_j [\lambda - 1] u_j - \Psi_{vj} u_j (1 - \alpha) - \Psi_{hj} u_j \omega \delta + \pi_j [u_{j-} - u_j] \quad (50)$$

where $\pi_j = \tilde{\pi}_j e^{\theta_j}$ is the transition intensity under the risk-neutral measure. The last term on the right-hand side captures the effect of a state switch, in which case firm value goes from u_j to u_{j-} . Differentiating the above equation with respect to y_j gives the optimal production quantity and selling price in each state:

$$y_j = \left(\frac{\Gamma_j (1 - \beta)}{1 + \sigma \eta_j \rho} \right)^{\frac{1}{\beta}} \Rightarrow p_j = \Gamma_j y_j^{-\beta} = \frac{1 + \sigma \eta_j \rho}{1 - \beta}.$$

Similarly, differentiating equation (50) with respect to z_j gives the optimal innovation rate in each state:

$$z_j = \frac{\phi}{\zeta} (\lambda \varphi - 1) u_j.$$

Plugging back the expressions for z_j and y_j into equation (50) gives the valuation equation of the initiator.

Consider now the dynamics of the exploiters. Following arguments similar to those in Section 3, their scaled value satisfies the following equation:

$$rx_j = \max_{y_{Xj} \geq 0} \frac{\omega}{1 - \omega \delta} y_{Xj} (p_{Xj} - 1 - \eta_j \rho \sigma_X) - (\phi z_j + \Psi_{vj}) (1 - \alpha_X) x_j - \omega \delta \Psi_{hj} x_j + \pi_j [x_{j-} - x_j], \quad (51)$$

where the last term on the right-hand side captures the effect of a state switch, in which case firm value goes from x_j to x_{j-} . Maximizing the above equation with respect to y_{Xj} gives the optimal production rate in each state:

$$y_{Xj} = \left(\frac{\Gamma_j (1 - \beta)}{1 + \eta_j \rho \sigma_X} \right)^{\frac{1}{\beta}}$$

which we plug back into equation (51) to get the valuation equation of the exploiter.

Finally, the scaled entrant value satisfy the following equation:

$$rw_j = \max_{v_j, h_j} - \eta_j \rho_W \sigma_W - \frac{\zeta_v}{2} v_j^2 - \frac{\zeta_h}{2} h_j^2 + \phi_v v_j [\Lambda u_j - w_j] + \phi_h h_j [\omega x_j - w_j] \quad (52)$$

$$+ \phi z_j (\lambda \varphi - 1) (w_j - \kappa) + \Psi_{v_j}^- (\Lambda - 1) (w - \kappa) - \Psi_{h_j}^- (w_j - \kappa) \omega \delta + \pi_j [w_{j-} - w_j].$$

The terms in this equation admit an interpretation similar to equation (20), and the last term on the right-hand side captures the effect of a state switch. In each state, the optimal rate of vertical innovation satisfies:

$$v_j = \frac{\phi_v}{\zeta_v} [\Lambda u_j - w_j], \quad (53)$$

whereas the optimal rate of horizontal innovation satisfies:

$$h_j = \frac{\phi_h}{\zeta_h} [\omega x_j - w_j]. \quad (54)$$

We plug back these expressions into equation (52) to obtain the valuation equation for the entrants.

In each state, the rate of creative destruction and the rate of horizontal displacement satisfy $\Psi_{v_j} = \mu_j \phi_v v_j$ and $\Psi_{h_j} = \mu_j \phi_h h_j$, and the free-entry condition $w_j = \kappa$ holds.

A.2.2 Proof of Proposition 4

Vertical innovation Following steps similar to those in Appendix A.1.2, it is possible to show that the initiator value in each state j satisfy:

$$u_j = \frac{\kappa}{\Lambda} + \frac{1}{\Lambda} \sqrt{\frac{2\zeta_v (r\kappa + \eta_j \rho_W \sigma_W)}{\phi_v^2}}. \quad (55)$$

As, realistically, we assume that $\eta_B > \eta_G$, then the value of the imitator is greater in state B . Using (55) into the expressions of z_j and d_j gives

$$z_j = \frac{\phi(\lambda - 1)}{\zeta} \left[\frac{\kappa}{\Lambda} + \frac{1}{\Lambda} \sqrt{\frac{2\zeta_v (r\kappa + \eta_j \rho_W \sigma_W)}{\phi_v^2}} \right] \quad (56)$$

as well as the optimal (vertical) innovation rate of active entrants:

$$v_j = \frac{\phi_v}{\zeta_v} \sqrt{\frac{2\zeta_v (r\kappa + \eta_j \rho_W \sigma_W)}{\phi_v^2}} = \sqrt{\frac{2(r\kappa + \eta_j \rho_W \sigma_W)}{\zeta_v}}. \quad (57)$$

Hence, the first part of the claim in Proposition 4 follows.

Consider now the the rate of creative destruction. In the two states, it satisfies:

$$\Psi_{v_j}(\eta_j, \eta_{j-}) = \frac{1}{1 - \alpha} \left(\frac{\phi^2}{2\zeta} u_j (\lambda \varphi - 1)^2 - r + \frac{\Upsilon(\eta_j)}{u_j} + \pi_j \frac{(u_{j-} - u_j)}{u_j} \right) \quad (58)$$

Let us start by considering the case in which Γ_j does not vary across states—i.e., $\Gamma_G = \Gamma_B \equiv \Gamma$. Now, express $\eta_B = \eta_G + \Delta$, with $\Delta \geq 0$. If $\Delta = 0$, $\eta_B = \eta_G$, and we are back to the one-state case,

meaning that $\Psi_{vB} = \Psi_{vG}$ —basically, there is no variation across the two states. Conversely, when $\Delta > 0$, $\Psi_{vB} \neq \Psi_{vG}$. To study the relative magnitude of creative destruction in the two states, we next define the function $F(\Delta) = \Psi_{vB}(\Delta) - \Psi_{vG}(\Delta)$ for $\Delta \geq 0$. As just discussed, $F(0) = 0$ holds. Using equation (58), we study $F'(\Delta)$. Let us also express u_B and u_G as a function of Δ . By calculations, we find that

$$F'(\Delta) = -\frac{\rho\sigma}{(1-\alpha)u_B} \left(\frac{\Gamma(1-\beta)}{1+(\eta_G+\Delta)\rho\sigma} \right)^{\frac{1}{\beta}} - \frac{u'_B}{(1-\alpha)} \left(\frac{\pi_G}{u_G} + \frac{\pi_B u_G}{u_B^2} \right) - \left[\beta \left(\frac{(1-\beta)}{1+(\eta_G+\Delta)\rho\sigma} \right)^{\frac{1}{\beta}-1} \Gamma^{\frac{1}{\beta}} - \frac{\phi^2(\lambda\varphi-1)^2 u_B^2}{2\zeta} \right] \frac{u'_B(\Delta)}{(1-\alpha)u_B^2}$$

with $u'_B(\Delta) = \frac{\zeta_v \rho_W \sigma_W}{\Lambda \phi_v \sqrt{2\zeta_v [r\kappa + (\eta_G + \Delta)\rho_W \sigma_W]}} > 0$. The term $\left[\beta \left(\frac{(1-\beta)}{1+(\eta_G+\Delta)\rho\sigma} \right)^{\frac{1}{\beta}-1} \Gamma^{\frac{1}{\beta}} - \frac{\phi^2(\lambda\varphi-1)^2 u_B^2}{2\zeta} \right]$ is positive under our assumption that the initiator's expected net cash flow is positive. As a result, all the terms in $F'(\Delta)$ are negative. Thus, the function F is zero at $\Delta = 0$ and decreases for $\Delta > 0$, meaning that $\Psi_{vB}(\Delta) < \Psi_{vG}(\Delta)$ if $\eta_B > \eta_G$. That is, the rate of creative destruction is procyclical.

Consider now the case $\Gamma_G > \Gamma_B$. If $\Delta = 0$, then $u_B = u_G$, $z_B = z_G$, and $v_B = v_G$, as these quantities do not depend on Γ_j (see equations (55), (56), and (57)). Consider again the function $F(\Delta)$ defined above. Let us first evaluate this function for $\Delta = 0$. Using equation (58), we have that $F(0) = \frac{1}{(1-\alpha)u_B(0)} \beta \left(\frac{(1-\beta)}{1+\eta_G\rho\sigma} \right)^{\frac{1}{\beta}-1} \left(\Gamma_B^{\frac{1}{\beta}} - \Gamma_G^{\frac{1}{\beta}} \right)$, where we have used that $u_B(0) = u_G(0)$. As $\Gamma_G > \Gamma_B$ by assumption, then $F(0) < 0$. As $F'(\Delta) < 0$ following the steps above, then $\Psi_{vB}(\Delta) < \Psi_{vG}(\Delta)$ for the case $\Gamma_G > \Gamma_B$ too.

Recall that $\Psi_{vj} = \mu_{vj} \phi_v v_j$. As shown above, $\Psi_{vB} - \Psi_{vG} < 0$ and $v_B > v_G$. Thus, for $\Psi_{vB} - \Psi_{vG} = \phi_v [\mu_B v_B - \mu_G v_G] < 0$ to hold, it must be that $\mu_B < \mu_G$. Thus, the mass of active entrants is also procyclical. Moreover, using the expression for v_j gives:

$$\Psi_{vB} - \Psi_{vG} = \phi_v \left[\mu_B \sqrt{\frac{2(r\kappa + (\eta_G + \Delta)\rho_W \sigma_W)}{\zeta_v}} - \mu_G \sqrt{\frac{2(r\kappa + \eta_G \rho_W \sigma_W)}{\zeta_v}} \right] \quad (59)$$

Note that the first square root is greater than the second, so we express $\sqrt{\frac{2(r\kappa + (\eta_G + \Delta)\rho_W \sigma_W)}{\zeta_v}} = A \sqrt{\frac{2(r\kappa + \eta_G \rho_W \sigma_W)}{\zeta_v}}$, with $A > 1$. Moreover, as noticed above, $\mu_B < \mu_G$, so we can express $\mu_G = B \mu_B$, with $B > 1$. Then, we have

$$\Psi_{vB} - \Psi_{vG} = \phi_v \mu_B \sqrt{\frac{2(r\kappa + \eta_G \rho_W \sigma_W)}{\zeta_v}} [A - B]. \quad (60)$$

As $\Psi_{vB} - \Psi_{vG} < 0$, then it must be that $A < B$, meaning that the mass of active entrants μ_j (the extensive margin) varies more than v_j (the intensive margin) for a given variation in Δ .

Horizontal innovation Consider now the case in which entrants pursue horizontal innovation only. Following steps similar to those in Appendix A.1.3, it is possible to solve for x_j from the

entrants' HJB equation simply by solving for the optimal h investment and imposing the free entry condition. We get

$$x_j = \frac{\kappa}{\omega} + \frac{1}{\omega} \sqrt{\frac{2\zeta_h(r\kappa + \eta_j\rho_W\sigma_W)}{\phi_h^2}}. \quad (61)$$

Using this expression into h_j gives

$$h_j = \sqrt{\frac{2(r\kappa + \eta_j\rho_W\sigma_W)}{\zeta_h}}. \quad (62)$$

Because $\eta_B > \eta_G$, then $h_B > h_G$. The claims in Proposition 4 then follow.

A.3 Proof of the results in Section 5

A.3.1 Proof of Proposition 5

We consider the definition of the risk premium of the initiator, $\mathcal{R}_{U,j}$, as shown in equation (35). Using the expression for u_j derived in equation (55), the derivative of $\mathcal{R}_{U,j}$ with respect to ϕ_v is thus given by:

$$\frac{\partial \mathcal{R}_{U,j}}{\partial \phi_v} = \frac{\eta_j \Lambda \rho \sigma y_j \sqrt{2\zeta_v(\eta_j \rho_W \sigma_W + \kappa r)}}{(\kappa \phi_v + \sqrt{2\zeta_v(\eta_j \rho_W \sigma_W + \kappa r)})^2} + \frac{\kappa \sqrt{2\zeta_v} (\sqrt{(\eta_{-j} \rho_W \sigma_W + \kappa r)} - \sqrt{(\eta_j \rho_W \sigma_W + \kappa r)})}{(e^{\theta_j} - 1)^{-1} \tilde{\pi}_j^{-1} (\kappa \phi_v + \sqrt{2\zeta_v(\eta_j \rho_W \sigma_W + \kappa r)})^2},$$

where the first term is strictly positive for any parameter value, and the second term is also positive in both states. In the good state, the numerator of the second term is positive given $\eta_B > \eta_G$, whereas $e^{\theta_G} - 1 > 0$ in the denominator. In the bad state, the numerator is negative given $\eta_B > \eta_G$, whereas $e^{\theta_B} - 1 < 0$ in the denominator. It follows that $\mathcal{R}_{U,j}$ is increasing in ϕ_h .

Next we calculate the derivative of $\mathcal{R}_{U,j}$ with respect to Λ :

$$\frac{\partial \mathcal{R}_{U,j}}{\partial \Lambda} = \frac{\eta_j \phi_v \rho \sigma y_j}{(\kappa \phi_v + \sqrt{2\zeta_v(\eta_j \rho_W \sigma_W + \kappa r)})},$$

which is strictly positive for any parameter value. It follows that $\mathcal{R}_{U,j}$ is increasing in Λ .

Consider next the risk premium of the exploiter, $\mathcal{R}_{X,j}$ defined in equation (36). Using the expression for x_j in equation (61), we obtain:

$$\frac{\partial \mathcal{R}_{X,j}}{\partial \phi_h} = \frac{\sqrt{2\kappa\tilde{\pi}_j} (\sqrt{\zeta_h(\eta_j \rho_W \sigma_W + \kappa r)} - \sqrt{\zeta_h(\eta_{-j} \rho_W \sigma_W + \kappa r)})}{(1 - e^{\theta_j})^{-1} (\kappa \phi_h + \sqrt{2\zeta_h(\eta_j \rho_W \sigma_W + \kappa r)})^2} + \frac{\eta_j \sqrt{2\zeta_h(r\kappa + \eta_j \rho_W \sigma_W)}}{(\rho_X \sigma_X y_{X,j})^{-1} x_j^2 (1 - \delta\omega) \phi_h^2},$$

which is strictly positive for any parameter value, given $\eta_B > \eta_G$, $e^{\theta_G} - 1 > 0$ and $e^{\theta_B} - 1 < 0$. It follows that $\mathcal{R}_{X,j}$ is increasing in ϕ_h as stated in Proposition (5).

We next consider the derivative of $\mathcal{R}_{X,j}$ with respect to ω :

$$\frac{\partial \mathcal{R}_{U,j}}{\partial \omega} = \frac{\eta_j \rho \sigma_X y_{X,j} (\kappa \phi_h + \sqrt{2\zeta_h(\eta_j \rho_W \sigma_W + \kappa r)})}{x_j^2 \omega (1 - \omega\delta)^2 \phi_h} + \frac{\eta_j \rho \sigma_X y_{X,j}}{x_j (1 - \omega\delta)},$$

which is strictly positive, proving that $\mathcal{R}_{X,j}$ is increasing in ω . Similarly, the derivative of $\mathcal{R}_{X,j}$ with respect to δ equals:

$$\frac{\partial \mathcal{R}_{U,j}}{\partial \delta} = \frac{\eta_j \rho \sigma_X \omega^3 y_{X,j} \phi_h}{(1 - \delta \omega)^2 (\kappa \phi_h + \sqrt{2 \zeta_h (\eta_j \rho_W \sigma_W + \kappa r)})},$$

which is strictly positive, proving that $\mathcal{R}_{X,j}$ is increasing in δ . Proposition (5) then follows.

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Table 1: BASELINE PARAMETERS.

Parameter	Description	Value
r	Risk-free rate	0.01
η	Market price of risk (one state)	0.30
ϕ	Poisson coefficient (initiator)	1.00
ϕ_v	Poisson coefficient (entrant, vertical)	1.00
ϕ_h	Poisson coefficient (entrant, horizontal)	1.00
ζ	R&D cost coefficient (initiator)	0.60
ζ_v	R&D cost coefficient (entrant, vertical)	6.00
ζ_h	R&D cost coefficient (entrant, horizontal)	0.08
λ	Vertical jump (initiator)	1.055
Λ	Vertical jump (entrants)	1.12
φ	Horizontal jump (initiator)	1.14
ω	Horizontal jump (entrants)	0.25
δ	Obsolescence due to horizontal innovations	0.20
β	Inverse of price elasticity of demand	0.13
Γ	Demand shift parameter	1.00
α	Recovery in liquidation (initiator)	0.60
α_X	Recovery in liquidation (exploiter)	0.85
σ	Coefficient of cash flow volatility (initiator)	0.80
σ_X	Coefficient of cash flow volatility (exploiter)	0.70
σ_W	Cash flow volatility (entrant)	0.20
ρ	Correlation with aggregate shocks (initiator and exploiter)	0.55
ρ_W	Correlation with aggregate shocks (entrant)	0.20
κ	Entry cost	0.015

Table 2: INNOVATION IN THE VERTICAL AND HORIZONTAL DIMENSION. This table reports the model endogenous quantities for the corner case in which entrants invest in vertical innovation only, for the corner case in which entrants invest in horizontal innovation only, and in the full case featuring both vertical and horizontal innovation. The top panel illustrates the case in which $\omega = 0.25$ (as in the baseline parameterization), whereas the bottom panel illustrates the case in which ω is higher and equal to 0.45.

	Vertical only	Horizontal only	Full case (both)
$\omega = 0.25$ (Baseline)			
z	0.120	–	0.116
v	0.064	–	0.062
h	–	–	0.141
μ	2.559	–	2.112
Ψ_v	0.163	–	0.130
Ψ_h	–	–	0.297
$\omega = 0.45$			
z	0.120	0.248	0.071
v	0.064	–	0.037
h	–	0.551	0.451
μ	2.559	0.918	1.881
Ψ_v	0.163	–	0.069
Ψ_h	–	0.506	0.849

Table 3: EQUILIBRIUM QUANTITIES. This table reports the quantities of interest for the case in which the market price of risk varies over time (first to third column) as well as when assuming that there is just one state of the economy in which the market price of risk is fixed at its two-state average. In the top panel, we assume that only the market price of risk η_j varies across the different states; in the bottom panel, we assume that the demand shift parameter Γ_j varies too.

	State G	State B	Average (two states)	One-state
Varying η_i only				
z	0.112	0.128	0.116	0.116
v	0.060	0.068	0.062	0.062
h	0.134	0.151	0.138	0.141
μ	2.873	0.170	2.260	2.107
Ψ_v	0.171	0.012	0.135	0.130
Ψ_h	0.385	0.026	0.304	0.296
\mathcal{I}	0.283	0.140	0.251	0.246
Varying η_i and Γ_i				
z	0.112	0.128	0.115	0.115
v	0.059	0.068	0.061	0.061
h	0.147	0.164	0.151	0.154
μ	3.224	0.160	2.529	2.341
Ψ_v	0.191	0.011	0.150	0.143
Ψ_h	0.475	0.026	0.373	0.360
\mathcal{I}	0.302	0.139	0.265	0.258